Neutrinos with Z_3 Symmetry and New Charged-Lepton Interactions

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(II) Broken $Z_3 \Rightarrow \Delta m^2_{sol} \neq 0 \ , \ \tan^2\theta_{sol} = 0.5$

(III) 3 Higgs doublets and new contributions to $\mu \to eee$, $\mu \to e\gamma$, etc.

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In the $(\nu_e, \nu_\mu, \nu_\tau)$ basis, consider

$$m_\nu = \underbrace{A\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{m_A} + \underbrace{B\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}}_{m_B} + \underbrace{C\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}}_{m_C}$$

then $$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

with $$\left.\begin{aligned} m_1 &= A - B \\ m_2 &= A - B + 3C \\ m_3 &= A + B \end{aligned}\right\} \quad \begin{aligned} \tan^2\theta_{sol} &= 0.5 \\ \sin^2 2\theta_{atm} &= 1 \end{aligned}$$

(I) normal hierarchy : $B = A$, $C \ll A$,

(II) inverted hierarchy : $B = -A$, $C \ll A$,

(III) near degeneracy : $C \ll B \ll A$.

<u>Note</u>: $\mathcal{U}_{e3} = 0$ is the consequence of

$$\mathcal{U}_2 \mathcal{M}_\nu \mathcal{U}_2^T = \mathcal{M}_\nu \ , \quad \mathcal{U}_2^2 = 1 \ ,$$

where $$\mathcal{U}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \ .$$

Since C is small in all cases, consider

$\mathcal{M}_\nu = M_A + M_B$, then

$$\mathcal{U}_B \mathcal{M}_\nu \mathcal{U}_B^T = \mathcal{M}_\nu \ , \quad \mathcal{U}_B^3 = 1 \ ,$$

where
$$\mathcal{U}_B = \begin{pmatrix} -\frac{1}{2} & -\sqrt{\frac{3}{8}} & -\sqrt{\frac{3}{8}} \\ \sqrt{\frac{3}{8}} & \frac{1}{4} & -\frac{3}{4} \\ \sqrt{\frac{3}{8}} & -\frac{3}{4} & \frac{1}{4} \end{pmatrix}$$

i.e. a new Z_3 symmetry has been discovered!

Origin of $\mathcal{M}_C$

$$\mathcal{L}_{eff} = \frac{f_{ij}}{\Lambda}(\nu_i\phi^0 - l_i\phi^+)(\nu_j\phi^0 - l_j\phi^+) + H.c.$$

with $\frac{f_{ij} v^2}{\Lambda} = C$ for all i, j

Symmetry: $\mathcal{U}_C(\mathcal{M}_A + \mathcal{M}_C)\mathcal{U}_C^T = \mathcal{M}_A + \mathcal{M}_C$,

$$\mathcal{U}_C^3 = 1, \quad \mathcal{U}_C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$\mathcal{U}_C + \mathcal{U}_2 \Rightarrow S_3$ symmetry

$\Lambda \sim 10^{16}$ to 10^{18} GeV $\Rightarrow C \sim 10^{-3}$ to 10^{-5} eV

$(\Delta m^2)_{sol} \sim 10^{-4}\ eV^2 \Rightarrow A - B + \frac{3C}{2} \sim 10^{-2}$ to 1 eV

$\Rightarrow$ consistent with all 3 solutions on p. 2

Origin of $\mathcal{M}_A + \mathcal{M}_B$

$$\mathcal{L}_Y = h_{ij}\left[\xi^0 \nu_i \nu_j - \xi^+\left(\frac{\nu_i l_j + l_i \nu_j}{\sqrt{2}}\right) + \xi^{++} l_i l_j\right]$$
$$+ f_{ij}^k \left(l_i \phi_j^0 - \nu_i \phi_j^-\right) l_k^c + H.c.$$

where $U_B^T h U_B = h$, $U_B^T f^k U_B = f^k$

$$\Rightarrow h = \begin{pmatrix} a-b & 0 & 0 \\ 0 & a & -b \\ 0 & -b & a \end{pmatrix}, \quad f^k = \begin{pmatrix} a_k - b_k & d_k & d_k \\ -d_k & a_k & -b_k \\ -d_k & -b_k & a_k \end{pmatrix}$$

$$\Rightarrow A = 2a\langle\xi^0\rangle, \quad B = 2b\langle\xi^0\rangle,$$

i.e. the structure of $\mathcal{M}_A + \mathcal{M}_B$ is preserved

Note: $\langle\xi^0\rangle \sim \frac{\mu v^2}{m_\xi^2}$ is naturally small

for $m_\xi^2 > 0$ and large.

Z_3 is softly broken by $m_1^2 \phi_1^\dagger \phi_1$.

This allows the solution $v_{1,2} \ll v_3$,

then the hierarchy of m_e, m_μ, m_τ is

understood from $d_k \ll b_k \ll a_k$

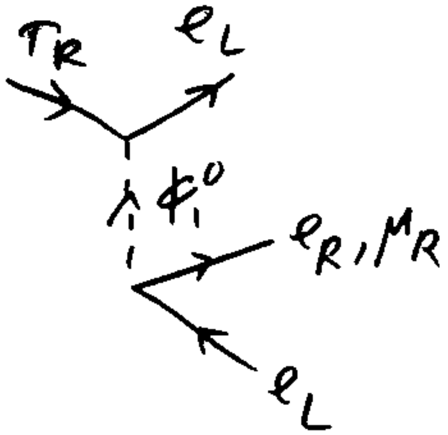
(which by itself does not break Z_3)

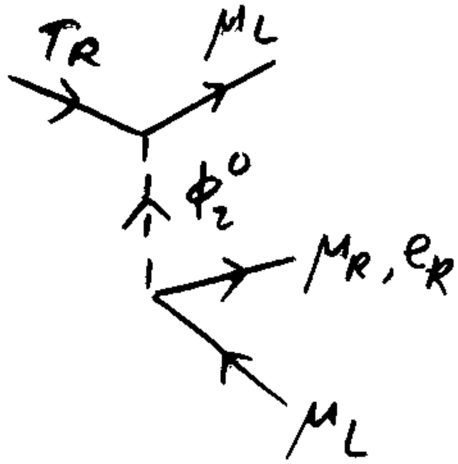
$$V_L \mathcal{M}_\ell \mathcal{M}_\ell^\dagger V_L^\dagger = \begin{pmatrix} m_e^2 & & \\ & m_\mu^2 & \\ & & m_\tau^2 \end{pmatrix}$$

$$\Rightarrow V_L \sim \begin{pmatrix} 1 & O\left(\frac{m_e}{m_\mu}\right) & O\left(\frac{m_e}{m_\tau}\right) \\ O\left(\frac{m_e}{m_\mu}\right) & 1 & O\left(\frac{m_\mu}{m_\tau}\right) \\ O\left(\frac{m_e}{m_\tau}\right) & O\left(\frac{m_\mu}{m_\tau}\right) & 1 \end{pmatrix}$$

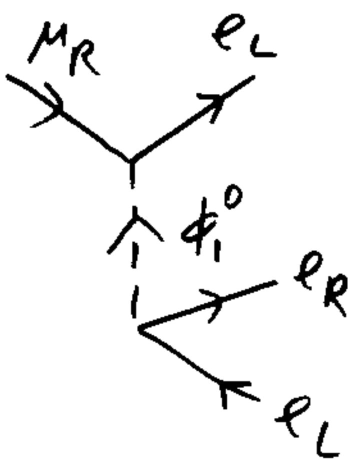
$$\Rightarrow U_{e3} \sim O\left(\frac{m_e}{m_\mu}\right),$$

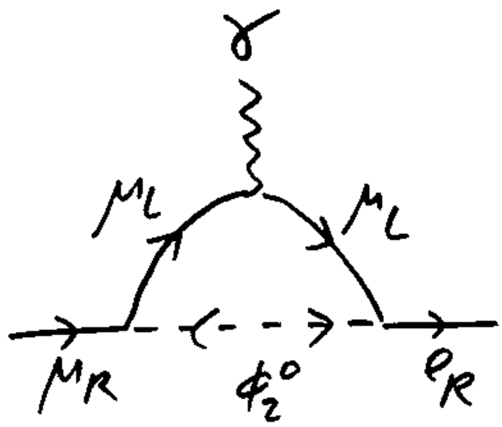
FCNC are nonzero but small.





$$B \sim \left(\frac{m_\mu^2 m_\tau^2}{m_{1,2}^4}\right) B(\tau \to \mu\nu\nu) \simeq 6.1 \times 10^{-11} \left(\frac{100\,\mathrm{GeV}}{m_{1,2}}\right)^4$$





$$B(\mu \to eee) \sim \frac{m_\mu^4}{m_1^4} \simeq 1.2 \times 10^{-12} \left(\frac{100\,\mathrm{GeV}}{m_1}\right)^4$$

$$B(\mu \to e\gamma) \sim \frac{3\alpha}{8\pi} \frac{m_\tau^4}{m_{eff}^4} < 1.2 \times 10^{-11}$$

$$\Rightarrow m_{eff} > 164\ \mathrm{GeV}$$

Suppose

$$m_\nu = \begin{pmatrix} a & d & e \\ d & b & f \\ e & f & c \end{pmatrix}$$

then $\tan^2\theta_{sol} = \left(\frac{1 - \sqrt{1+z^2}}{z} \right)^2$

where $z = \frac{2\sqrt{2}\,(d+e)}{2(f-a)+b+c}$.

e.g. $z = \begin{cases} 2\sqrt{2} \\ 2.2 \end{cases} \Rightarrow \tan^2\theta_{sol} = \begin{cases} 0.5 \\ 0.42 \end{cases}$

Also, $U_{e3} \simeq \frac{d-e}{2\sqrt{2}\,B}$.

<u>Conclusion</u>: $m_\nu = \underbrace{\begin{pmatrix} A-B & 0 & 0 \\ 0 & A & -B \\ 0 & -B & A \end{pmatrix}}_{Z_3 \text{ invariant}} + m_\nu'$