CPT VIOLATION:
Cosmological & Quantum-Gravity Effects
&
their Phenomenology

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NO-VE, December 2003

ESSENTIAL QUESTIONS

- **Are there theories which allow CPT breaking?**
- **How (un)likely is it that somebody someday finds CPT violation, and why?**
- **What formalism? How can we be sure of observing CPT Violation ? our current phenomenology is based on CPT invariance**
- How should we compare various "figures of merit" of CPT tests (direct mass measurement, K^0-$\overline{K}^0$ mass difference a la CPLEAR, Decoherence Effects, EPR states, neutrino mixing, electron g-2 and cyclotron frequency comparison, neutrino spin-flavour conversion (for Lorentz violating models) *etc.*)

OUTLINE

- WHAT IS CPT SYMMETRY.
- WHY CPT VIOLATION ?

  Theoretical models and ideas: **Quantum Gravity Models violating Lorentz symmetry and/or quantum coherence**:

  (i) space-time foam (local f.t., non-critical strings),

  (ii) (non supersymmetric) string-inspired standard model extension with Lorentz Violation.

  (iii) Loop Quantum Gravity

- HOW CAN WE DETECT CPT VIOLATION? :

  SENSITIVE PARTICLE PHYSICS PROBES:

  (i) Neutral Mesons: KAONS, B-MESONS, entangled states in ϕ and B factories.

  (ii) anti-matter factories: antihydrogen (precision spectroscopic tests on free and trapped molecules),

  (iii) Low energy atomic physics experiments.

  * *(iv) Neutrino Physics*

  (v) Astrophysical Tests (especially Lorentz-Invariance violation tests, via modified dispersion relations of matter probes *etc.*)

  *: *This talk.*

CPT THEOREM

C(harge) -P(arity=reflection) -T(ime reversal) INVARIANCE is a property of any quantum field theory in Flat space times which respects: (i) Locality, (ii) Unitarity and (iii) Lorentz Symmetry.

$$\Theta \mathcal{L}(x) \Theta^{\dagger} = \mathcal{L}(-x) \ ,$$

$$\Theta = CPT \ , \ \mathcal{L} = \mathcal{L}^{\dagger} \ (\text{Lagrangian})$$

Theories with HIGHLY CURVED SPACE TIMES , with space time boundaries of black-hole horizon type, may violate (ii) & (iii) and hence CPT.

E.g.: SPACE-TIME FOAMY SITUATIONS IN SOME QUANTUM GRAVITY MODELS.

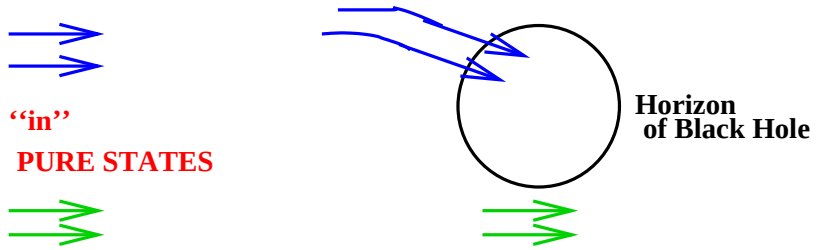
SPACE-TIME FOAM

Space-time MAY BE DISCRETE at scales $10^{-35}\ m$ (Planck) $\rightarrow$ LORENTZ VIOLATION (LV)? (and hence CPTV); also there may be ENVIRONMENT of GRAVITATIONAL d.o.f. INACCESSIBLE to low-energy experiments (non-propagating d.o.f., no scattering) $\rightarrow$ CPT VIOLATION (and may be LV)

FOAM AND UNITARITY VIOLATION

SPACE-TIME FOAM: Quantum Gravity SINGULAR Fluctuations (microscopic (Planck size) black holes etc) may imply "environment" $\rightarrow$ evolution of initially pure states to mixed ones:

SPACE–TIME FOAMY SITUATIONS
NON UNITARY (CPT VIOLATING) EVOLUTION
OF PURE STATES TO MIXED ONES



"out"
MIXED STATES

$|\ldots>$

ρ_{out} = density matrix

$= \mathrm{Tr}_{unobs} |\psi><\psi|$

modified temporal evolution of ρ:

$$\frac{d}{dt}\rho = i\,[\rho\,,H\,] + \Delta H(\rho)\,\rho$$

quantum mechanical terms — **quantum mechanics violating term**

$$\rho_{out} = \mathrm{Tr}_{\mathrm{unobs}}|out><out| = \$\ \rho_{in}$$

$\$ \neq SS^\dagger$, $S = e^{iHt}$ =scattering matrix \$ non invertible, unitarity lost in effective theory.

CPT VIOLATION (CPTV) AND \$ $\neq SS^\dagger$

A THEOREM BY R. WALD: **If \$ $\neq S\ S^\dagger$, then CPT is violated, at least in its strong form.**

PROOF:

Suppose CPT is conserved, then there exists unitary, invertible opearator Θ : $\Theta\overline{\rho}_{in} = \rho_{out}$

$$\rho_{out} = \$\ \rho_{in} \rightarrow \Theta\overline{\rho}_{in} = \$\ \Theta^{-1}\overline{\rho}_{out} \rightarrow \overline{\rho}_{in} = \Theta^{-1}\$\ \Theta^{-1}\overline{\rho}_{out}.$$

But $\overline{\rho}_{out} = \$\overline{\rho}_{in}$, hence :

$$\overline{\rho}_{in} = \Theta^{-1}\$\Theta^{-1}\ \$\ \overline{\rho}_{in}$$

BUT THIS IMPLIES THAT \$ HAS AN INVERSE- $\Theta^{-1}\$\Theta^{-1}$, IMPOSSIBLE (information loss), hence CPT MUST BE VIOLATED (at least in its strong form).

NB: My preferred way of CPTV by Quantum Gravity

COSMOLOGICAL CPTV?

(NM, hep-ph/0309221)

Recent Astrophysical Evidence for Dark Energy (acceleration of the Universe (SnIA), CMB anisotropies (WMAP...))

Best fit models of the Universe consistent with non-zero **cosmological constant** $\Lambda \neq 0$ (de Sitter)

Λ-universe will eternally accelerate, as it will enter in an inflationary phase again: $a(t) \sim e^{\sqrt{\Lambda/3}t}$, $t \to \infty$, there is **cosmological Horizon**.

Horizon implies incompatibility with S-matrix: no proper definition of asymptotic state vectors, environment of d.o.f. crossing the horizon (c.f. dual picture of black hole, now observer is inside the horizon).

Theorem by Wald on $-matrix and CPTV: CPT is violated due to $\Lambda > 0$:

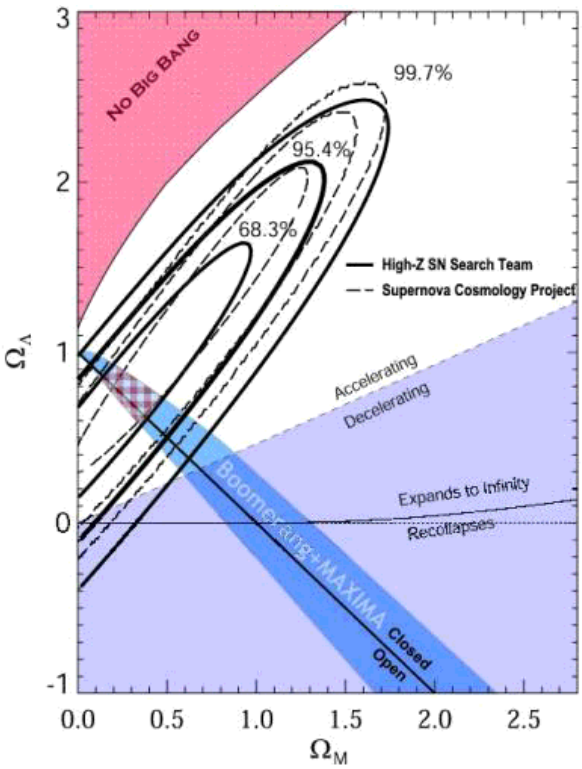
$$\partial_t \rho = [\rho, H] + \mathcal{O}(\Lambda/M_P^3)\rho$$

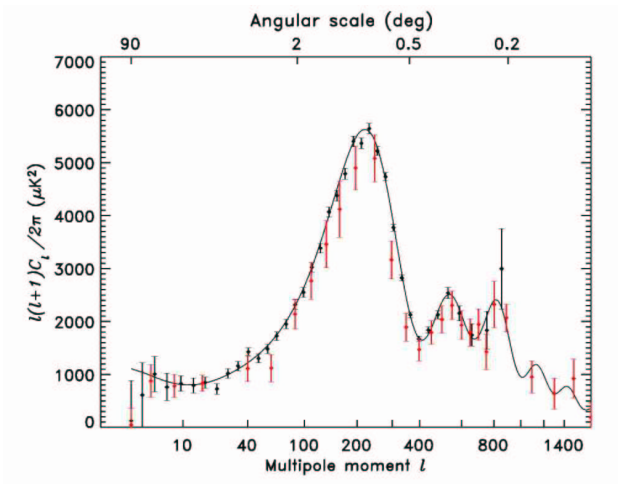
Tiny cosmological CPTV effects, but detected through Universe acceleration!

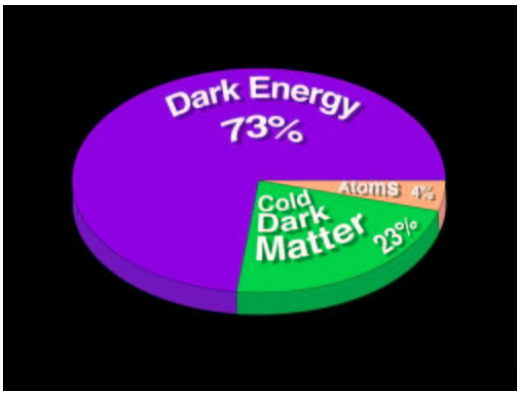
Issue: Quantize de Sitter space as an open system? or use Relaxation models for Dark Energy, where S-matrix is OK?.

Evidence for Dark Energy

WMAP improved results on CMB: $\Omega_{\rm total} = 1.02 \pm 0.02$, high precision measurement of secondary (two more) acoustic peaks (c.f. new determination of Ω_b). Agreement with Snla Data. Best Fit : $\Omega_\Lambda = 0.73$, $\Omega_{\rm Matter} = 0.23$







LORENTZ-VIOLATION AND CPT:

STANDARD MODEL EXTENSION (SME)

V.A. Kostelecký, R. Bluhm, D. Colladay, R. Potting, N. Russell

In this case Lorentz symmetry is violated and hence CPT, but no quantum decoherence or unitarity loss.

String theory (non supersymmetric) $\rightarrow$ Tachyonic instabilities, coupling with tensorial fields (gauge etc), $\rightarrow < A_\mu > \neq 0$, $< T_{\mu_1 \ldots \mu_n} > \neq 0$,

Spontaneous breaking of Lorentz symmetry by (exotic) string vacua

MODIFIED DIRAC EQUATION in SME: for FREE Hydrogen H (anti-hydrogen $\overline{H}$): spinor ψ reps. electron (positron) with charge $q = -|e| (q = |e|)$ around a proton (antiproroton) of charge $-q$:

$$(i\gamma^\mu D^\mu - M - a_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu - \\ -\frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu} + i c_{\mu\nu} \gamma^\mu D^\nu + i d_{\mu\nu} \gamma_5 \gamma^\mu D^\nu) \psi = 0$$

where $D_\mu = \partial_\mu - q A_\mu$, $A_\mu = (-q/4\pi r, 0)$ Coulomb potential. CPT & Lorentz violation: a_μ , b_μ .

Lorentz violation only: $c_{\mu\nu}$, $d_{\mu\nu}$, $H_{\mu\nu}$

ORDER OF MAGNITUDE of CPTV

Tiny cosmological (global) CPTV effects may be much smaller than QG (local) space-time effects (foam etc).

Naively, Quantum Gravity (QG) has a dimensionful constant: $G_N \sim 1/M_P^2$, $M_P = 10^{19}$ GeV. Hence, CPT violating and decoherening effects may be expected to be suppressed by E^3/M_P^2 , where E is a typical energy scale of the low-energy probe. This would be practically undetectable in neutral mesons, but neutrinons might be sensitive ! (e.g. modified dispersion relations (m.d.r.) for ultrahigh energy ν from GRB's (Ellis, NM, Nanopoulos, Volkov)) Also in some astrophysical cases, e.g. Crab Nebula or Vela pulsar synchrotron radiation constraints electron m.d.r. of this order (Jacobson, Liberati, Mattingly, Ellis, NM, Sakharov)

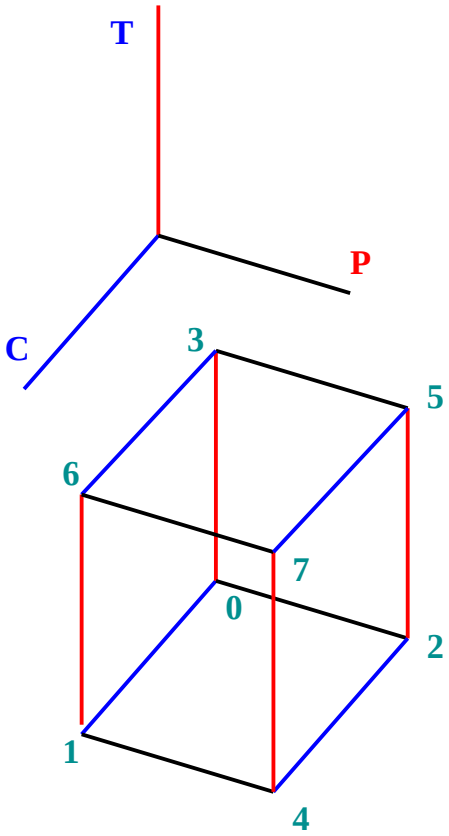
HOWEVER: RESUMMATION & OTHER EFFECTS in theoretical models may result in much larger effects of order: $\frac{E^2}{M_P}$.
(This happens, e.g., loop gravity, some stringy models of QG involving open string excitations ...)

SUCH LARGE EFFECTS ARE definitely ACCESSIBLE/FALSIFIABLE BY CURRENT AND IMMEDIATE FUTURE EXPERIMENTS.

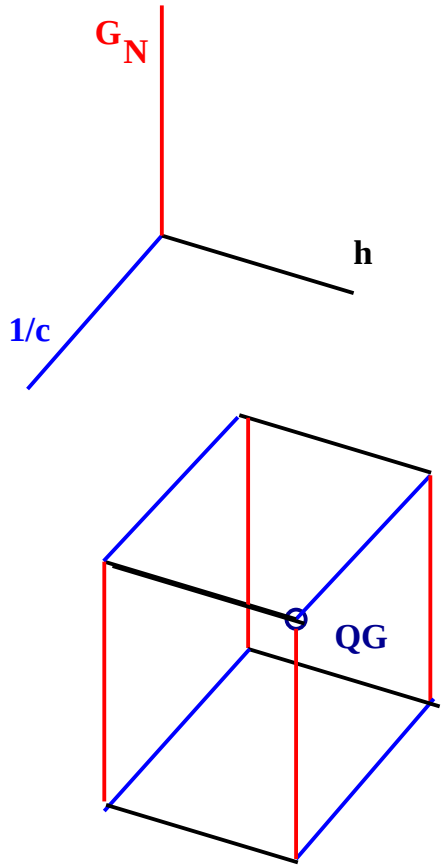
PHENOMENOLOGY of CPTV:

MNEMONIC CUBES

OKUN'S CUBE



PENROSE'S



CPT: 0, 4, 5, 6 even ; 1, 2, 3, 7 odd

QG MAY VIOLATE CPT ?

PHENOMENOLOGY of CPTV:

HOW CAN WE DETECT LORENTZ (and CPT) VIOLATION? :

Direct SME Tests & Modified Dispersion relations (M.D.R.)

Many LV Models of QG predict modified dispersion relations (d.r.) for matter probes, inclusive of ν (Amelino-Camelia, Ellis, NM, Nanopoulos, Sarkar, Volkov, Gambini, Pullin, ...).

This leads to one class of tests using ν: each mass eigenstate has QG deformed dispersion relations, may be the same for all flavours, may be not:

$E^2 = \vec{k}^2 + m_i^2 + f_i(E, M_{qg}, \vec{k})$, e.g. $f_i = \sum_\alpha C_\alpha \vec{k}^2 (\frac{|\vec{k}|}{M_{qg}})^\alpha$.

Stringent bounds on f_i from oscillation experiments.

GENERIC TESTS of M.D.R. &/OR S.M.E.

(i) astrophysics tests - arrival time fluctuations for photons (model independent analysis of astrophysical GRB data)

(ii) Nuclear/Atomic Physics precision measurements (clock comparison, spectroscopic tests on free and trapped molecules, quadrupole moments *etc*).

(iii) antihydrogen (precision spectroscopic tests on free and trapped molecules: e.g. $1S \to 2S$ forbidden transitions),

*(iii) Neutrino mixing, spin-flavour conversion.

PLANCK SCALE LV-SME BOUNDS

LOW-ENERGY ATOMIC PHYSICS EXPERIMENTS:

LEADING ORDER BOUNDS

EXPER.	SECTOR	PARAMS. (J=X,Y)	BOUND (GeV)
Penning Trap	electron	$\bar{b}_J^e$	5×10^{-25}
Hg−Cs clock comparison	electron	$\bar{b}_J^e$	10^{-27}
	proton	$\bar{b}_J^p$	10^{-27}
	neutron	$\bar{b}_J^n$	10^{-30}
H Maser	electron	$\bar{b}_J^e$	10^{-27}
	proton	$\bar{b}_J^p$	10^{-27}
spin polarized matter	electron	$\bar{b}_J^e / \bar{b}_Z^e$	$10^{-29} / 10^{-28}$
He−Xe Maser	neutron	$\bar{b}_J^n$	10^{-31}
Muonium	muon	$\bar{b}_J^\mu$	2×10^{-23}
Muon g−2	muon	$\bar{b}_J^\mu$	5×10^{-25} (estimated)

X,Y.Z celestial equatorial coordinates $\bar{b}_J = b_3 - m d_{30} - H_{12}$

(Bluhm, hep−ph/0111323)

Neutrinos & SME

SME-LV+CPTV (phenomenological) model for ν (Kostelecký & Mewes 20003)

$$\mathcal{L}_{SME} \ni \frac{1}{2} i \overline{\psi}_{a,L} \gamma^\mu D_\mu \psi_{a,L} - (a_L)_{\mu ab} \overline{\psi}_{a,L} \gamma^\mu \psi_{b,L} + \frac{1}{2} i (c_L)_{\mu\nu ab} \overline{\psi}_{a,L} \gamma^\mu D^\nu \psi_{b,L}$$

a, b flavour indices, No ν mass differences.

Presence of LV induces directional dependence (sidereal effects)!

Effective Hamiltonian:

$$(H_{\rm eff})_{ab} = |\vec{p}| \delta_{ab} + \frac{1}{|\vec{p}|} ((a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu)_{ab}$$

NB: ν Oscillations now are controlled by (dimensionless) $a_L L$ & $c_L L E$ (L=oscill. length). Contrast conventional case: $\Delta m^2 L/E$

Imporant SME feature: despite CPTV, oscillation probs $P_{\nu_x \to \nu_y} = P_{\bar{\nu}_x \to \bar{\nu}_y}$ (if no mass differences).

Bind LV+CPTV SME experimentally. E.g.: High energy long baseline expts: no evidence for $\nu_{e,\mu} \to \nu_\tau$ at $E \sim 100$ GeV , $L \sim 10^{-18}$ GeV^{-1} $\to a_L < 10^{-18}$ GeV, $c_L < 10^{-20}$.

For LSND anomaly: Mass-squared difference required: $\Delta m^2 = 10^{-19}$ GeV$^2 = 10^{-1}$ eV^2, $a_L \sim 10^{-18}$ GeV, $c_L \sim 10^{-17}$. Affect other expts. . No good for LSND.

Experimental Sensitivities for ν's

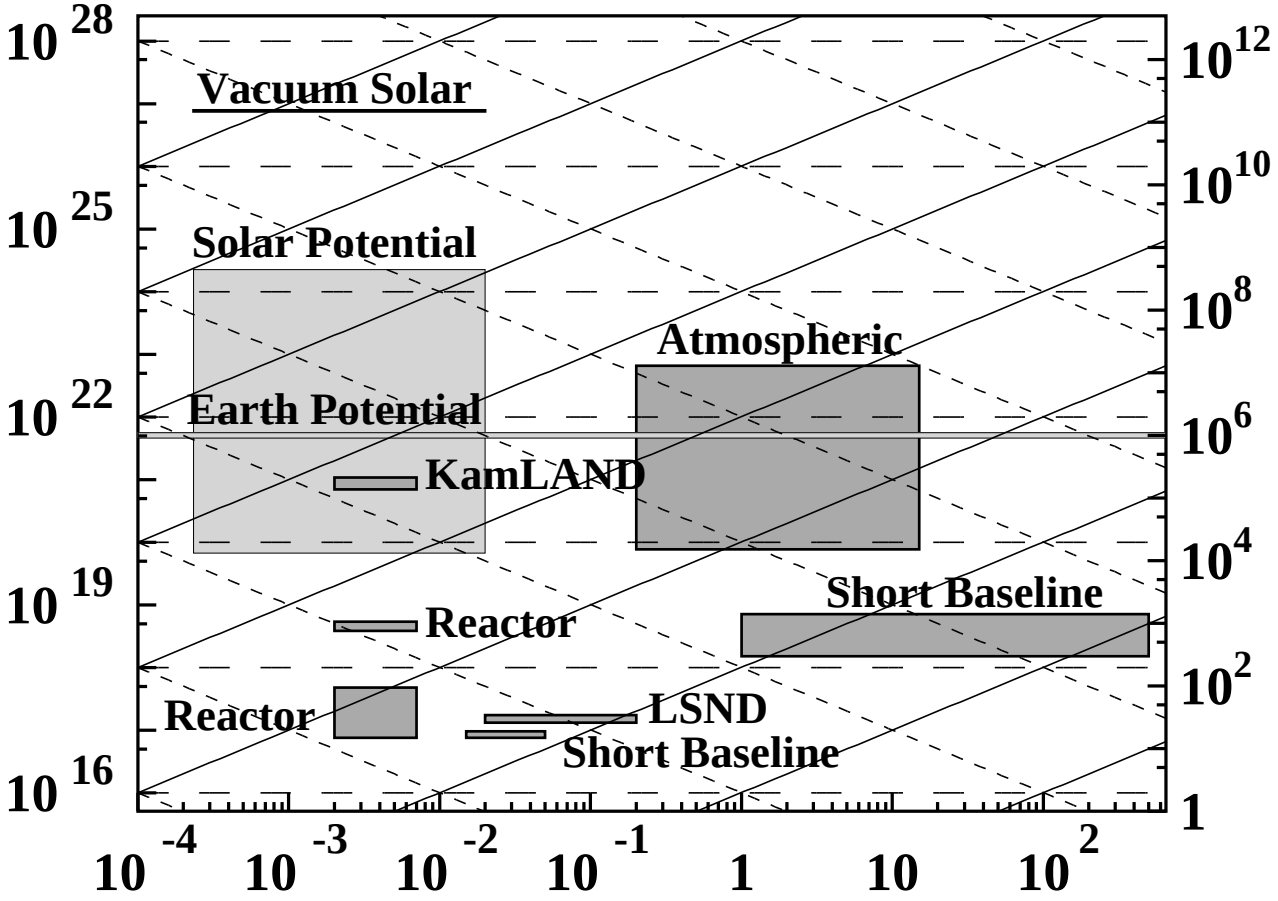


Figure 1: Approximate experimental sensitivities. Lines of constant L/E (solid), L (dashed), and LE (dotted) are shown, which give sensitivities to Δm^2, a_L, and c_L, respectively. (Kostelecky & Mews hep-ph/0308300)

Lorentz Invariance & ν spin-flavor conversion:

Loop Gravity Example: (Alfaro, Morales-Tecotl and Urrutia 2000, 2002) : $E_{\pm}^2 = A_p^2 p^2 + \eta p^4 \pm 2\lambda p + m^2$

$A_p = 1 + \kappa_1 \frac{\ell_P}{\mathcal{L}}$, $\eta = \kappa_3 \ell_P^2$, $\lambda = \kappa_5 \frac{\ell_P}{2\mathcal{L}^2}$, and $\mathcal{L} \sim E^{-1}$ or const., a characteristic scale.

Weak interaction Effects of ν propagating in a medium (Mikheyev Smirnov Wolfenstein effect): energy shift $\sqrt{2} G_F (2n_e - n_n)$. Plus interaction with external magnetic field B via radiatively induced magnetic moment μ: $\mu \overline{\psi} \sigma^{\mu\nu} F_{\mu\nu} \psi$.

SPIN-FLAVOUR CONVERSION:

$$i \frac{d}{dr} \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{eR} \\ \nu_{\mu R} \end{pmatrix} = \mathcal{H} \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{eR} \\ \nu_{\mu R} \end{pmatrix},$$

$\mathcal{H}$ = effective. Hamiltonian $\ni$ λ-effects.

(NB: For Majorana ν replace: $\nu_{iL} \to \nu_i$, $\nu_{iR} \to \overline{\nu}_i$).

Resonant Condition for Flavour-Spin-flip:

$$\nu_{eL} \to \nu_{\mu R} : \quad 2\lambda + \frac{\Delta m^2}{2p} \cos 2\theta - \sqrt{2} G_F n_e(r_{res}) = 0$$

$$\nu_{\mu L} \to \nu_{eR} : \quad 2\lambda - \frac{\Delta m^2}{2p} \cos 2\theta - \sqrt{2} G_F n_e(r_{res}) = 0$$

Lorentz Invariance & ν spin-flavor conversion:

Use above conditions to obtain bounds for λ, κ_i via:

$P_{\nu_{eL}\to\nu_{\mu R}} = \frac{1}{2}(1 - \cos 2\tilde{\theta}\cos 2\theta)$,

$\tan 2\tilde{\theta}(r) = \frac{4\mu B(r)E}{|\Delta m^2|\cos 2\theta - 4E\lambda + 2\sqrt{2}G_F E n_e(r)}$.

Assumptions: Reasonable profiles for solar $n_e \sim n_0 e^{-10.5r/R_\odot}$, $n_0 = 85 N_A \mathrm{cm}^{-3}$. Also: $\mu \sim 10^{-11}\mu_B$.

Upper bound on λ:

$\lambda \leq \frac{1}{2}\left(10^{-12} e^{-10.5 r_{res}/R_\odot}\,\mathrm{eV} + \frac{|\Delta m^2|}{2E}\right)$

- (i) $\mathcal{L}$=universal constant:

  Photon dispersion GRB, AGN tests: $\mathcal{L} \sim 10^{-18}$ eV^{-1}

  Best fit SOLAR ν-oscillations induced by MSW, use expt values of Δm^2, $\sin^2 2\theta$, and bind κ_i: $\kappa_5 < 10^{-25}$.

  From ATMOSHERIC oscillations, in particular LSND experiment, $\nu_\mu \to \nu_e$ fits the data with: $|\Delta m^2| \sim eV^2$, $\sin^2 2\theta \sim (0.2 - 3) \times 10^{-3}$, $E_{\max} \sim 10$ MeV, then $\kappa_5 < 10^{-17}$.

- (ii) $\mathcal{L} \sim p^{-1}$ (mobile scale) : From SOLAR oscillations, with $p \sim 1 - 10\mathrm{MeV}$. one gets $\kappa_5 = \mathcal{O}(1 - 100)$, natural range of values.

  From ATMOSHERIC oscillations,for the maximum ν $E \sim 10$ MeV, and $\mathcal{L} \sim E^{-1}$, one gets $\kappa_5 \sim 10^4$ (very weak bound). **CONCLUSION: (ii) is favoured.**

ν-mixing & modified Lorentz Invariance (LI):

A Peculiar way flavour ν states experience LI?

Deformed dispersion relations for ν flavor states (Blasone, Magueijo & Pires-Pacheco (BMP) (2003)): flavor states are *superposition of mass eigenstates* with standard d.r. of *different mass*.

$$E_e =< \nu_e|H|\nu_e >= \omega_{k,1}\cos^2\theta + \omega_{k,2}\sin^2\theta$$

$$E_\mu =< \nu_\mu|H|\nu_\mu >= \omega_{k,2}\cos^2\theta + \omega_{k,1}\sin^2\theta$$

$H|\nu_i >= \omega_i|\nu_i >$, $\omega_{k,i} = \sqrt{\vec{k}^2 + m_i^2}$. sum of two square roots in not in general a square root, hence modified d.r. for flavour states $\rightarrow$ LI?

Idea of BMP: Avoid using preferred frame by non-linear modified Lorentz transformations to ensure observer independence (Amelino-Camelia, Magueijo & Smolin):

$$E_i^2 f_i^2(E_i) - \vec{k}^2 g_i^2(E_i) = M_i^2 \quad i = e, \mu$$

Determine m(odified). d.r. $f_i(E_i, \theta, m_i), g_i(E_i, \theta, m_i), M_i(m_i, \theta)$ by comparing with $E_i = E_(\omega_i, m_i)$ above.

Identify non-linear Lorentz trnsf that leaves the m.d.r. invariant: $U \circ (E, \vec{k}) = (Ef, \vec{k}g)$.

ν-mixing & LI: Experiment

Test these ideas experimentally? β DECAY

Energy conservation in β-decay: $N_1 \to N_2 + e^- + \bar{\nu}_e$, (e.g. $N_1 =^3 H$, $N_2 =^3 He$)

Non-linear LI case:

$E_{N_1} = E_{N_2} + E + E_e f_e(E_e)$, ($E$=energy of e) vs

Preferred frame case: $E_{N_1} = E_{N_2} + E + E_e$,

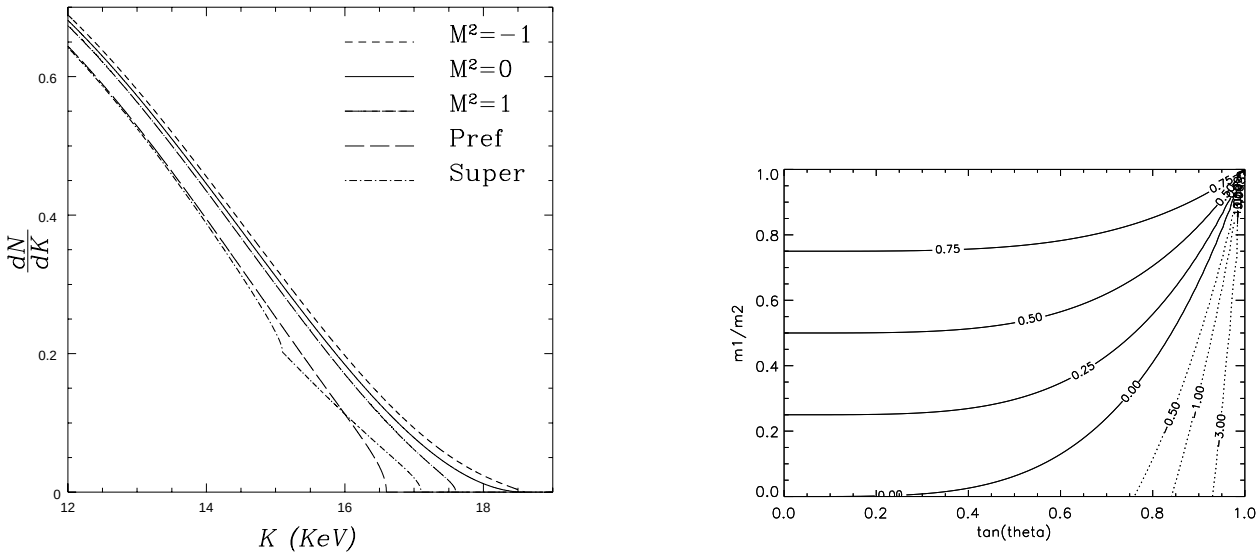


Figure 2: Left: Tail of tritium β-decay spectrum, for massless ν (solid) and for LI flavour states (dashed and dot-long-dashed). Also plotted is the preferred frame case. Right: Likelihood Contours of M^2 (in units of m_2^2) upon which β-decay depends.

PHENOMENOLOGY of CPTV (cont'd):

HOW CAN WE DETECT CPT VIOLATION? :
Departure from Quantum Mechanical evolution (QMV): QG Decoherence (c.f. open systems -gravitational 'environment')
Not necessarily Lorentz Violating (Millburn 2003)

SENSITIVE PARTICLE PHYSICS PROBES of QMV:

(i) neutral kaons and B-mesons (Ellis, Hagelin, Nanopoulos, Srednicki, (1984), + NM, Lopez (1992-95)), and ϕ-, B-factories (novel CPT tests for EPR states) (Bernabeu, NM, Papavassiliou 2003)

(ii) ultracold (slow) neutrons in Earth's gravitational field ?

* (iii) Neutrino flavour mixing ?

QG Decoherence and neutrino mixing

Quantum Gravity (QG) may induce oscillations between neutrino flavours independently of masses (Liu et al., 1997, Chang et al., 1998, Lisi et al., Benatti & Floreanini 2000).

$$\partial_t \rho = i[\rho, H] + \delta \not{H} \rho$$

where (Ellis, Hagelin, Nanopoulos, Srednicki 1984)

$$\delta \not{H}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -2\alpha & -2\beta & 0 \\ 0 & -2\beta & -2\gamma & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

for energy and lepton number conservation. and

$$\delta \not{H}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix}$$

if energy and lepton number violated, but flavour conserved (σ_1 Pauli matrix).

Positivity of ρ requires: $\alpha, \gamma > 0, \qquad \alpha\gamma > \beta^2$.

α, β, γ **violate CPT** (Ellis, NM, Nanopoulos 1992, Lopez + EMN 1995).

QG Decoherence and neutrino mixing

OSCILLATION PROBABILITIES

(i) For flavour conserving case:

Example: $\nu_e \to \nu_x$ $(x = \mu, \tau$ or sterile):

$$P_{\nu_e \to \nu_x} = \frac{1}{2} - \frac{1}{2} e^{-\gamma L} \cos^2 2\theta_v - \frac{1}{2} e^{-\alpha L} \sin^2 2\theta_v \cos(\frac{|m^2_{\nu_1} - m^2_{\nu_2}|}{2E_\nu} L)$$

Here L is oscillation length, θ_v is the mixing angle:

$\theta_v \neq 0 \longleftrightarrow$ massless ν's,

in mass basis: $|\nu_e >= \cos\theta_v |\nu_1 > + \sin\theta_v |\nu_2 >$,
$|\nu_\mu >= -\sin\theta_v |\nu_1 > + \cos\theta_v |\nu_2 >$.

NB: flavour oscillations even in massless case, due to γ compatible with flavour conserving formalism:

$< \nu_e|\sigma_1|\nu_e >= - < \nu_\mu|\sigma_1|\nu_\mu >= 2\sin\theta_v \cos\theta_v$.

NB: $P_{\nu_e \to \nu_x}(t \to \infty) = \frac{1}{n}$, for n generations.

QG Decoherence and neutrino mixing

OSCILLATION PROBABILITIES

(ii) For Energy and Lepton number conserving case:

Example: $\nu_e \to \nu_x$ ($x = \mu, \tau$ or sterile):

$$P_{\nu_e \to \nu_x} = \frac{1}{2}\sin^2 2\theta_v \left(1 - e^{-(\alpha+\gamma)L}\cos(\frac{|m^2_{\nu_1} - m^2_{\nu_2}|}{2E_\nu}L)\right)$$

assuming $\alpha, \beta, \gamma \ll \frac{|m^2_{\nu_1} - m^2_{\nu_2}|}{2E_\nu}$.

NB: $P_{\nu_e \to \nu_x}(t \to \infty) = \frac{1}{n}\sin^2 2\theta_v$, for n generations, **in contrast with case (i) above**.

EXPERIMENTAL SENSITIVITY

Use results (i) & (ii) to bind experimentally $\xi \equiv \{\alpha, \beta, \gamma\}$.

Theoretical Models Predictions vs. Experiment:

Optimistic: (Ellis, NM, Nanopoulos, ...)
$\xi \sim \xi_0(\frac{E}{\mathrm{GeV}})^n, n = 0, 2, -1.$

Pessimistic: (Adler 2000) $\xi \sim \frac{(\Delta m^2)^2}{E^2 M_{qg}}$,
($M_{qg} \sim M_P \sim 10^{19}$ GeV).

QG Decoherence and neutrino mixing

In some models of QG Decoherence, complete positivity of $\rho(t)$ for composite systems, such as ϕ or B mesons, may be imposed (Benatti & Floreannini). This results in ideal Markov environment, with:

$\alpha = \beta = 0, \gamma > 0.$

If this model assumed for ν (Lisi et all., PRL 85 (2000), 1166), then phenomenological parametrisation:

$\gamma = \gamma_0 (E/\mathrm{GeV})^n$, $n = 0, 2, -1.$

with E the neutrino energy.

From Atmosperic ν data $\rightarrow$ Bounds:

$n = 0$, $\gamma_0 < 3.5 \times 10^{-23}$ GeV

$n = 2$, $\gamma_0 < 0.9 \times 10^{-27}$ GeV (c.f. CPLEAR bound for Kaons: $\gamma < 10^{-21}$ GeV (PLB364 (1995) 239))

$n = -1$, $\gamma_0 < 2 \times 10^{-21}$ GeV.

NB: Tests on ν-mixing from Decoherence exhibit much greater sensitivity than neutral mesons.

QG Decoherence and neutrino mixing

FITTING THE DATA (Lisi et al. PRL 85 (2000), 1166)

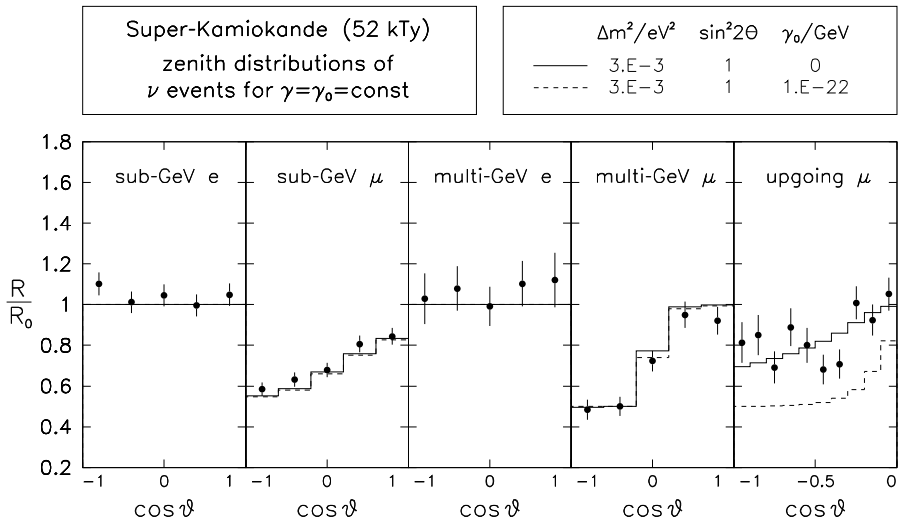


Figure 3: Effects of decoherence $(\gamma = \gamma_0 = \mathrm{const} \neq 0)$ on the distributions of lepton events as a function of the zenith angle ϑ

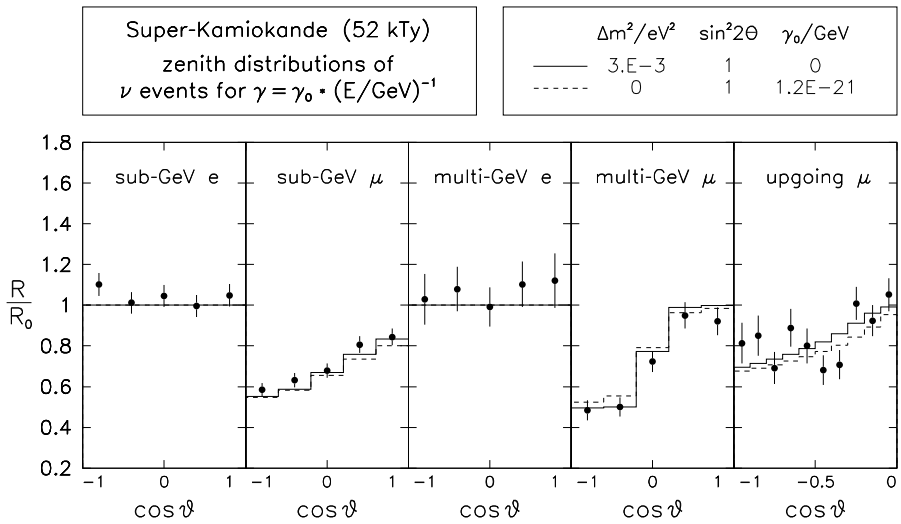


Figure 4: Best-fit scenarios for pure oscillations $(\gamma = 0)$ (solid line) and for pure decoherence with $\gamma \propto 1/E$ (dashed line).

PHENOMENOLOGY of CPTV (cont'd):

HOW CAN WE DETECT CPT VIOLATION? (Cont'd)

*Departure from Locality (and Neutrinos)

- (ν) Neutrino Physics Anomalies (LSND) and CPT Violation: Instead of Sterile neutrino introduce CPT violation (CPTV) in neutrino sector.
- Existing models (Barenboim and Lykken 2001) violate LOCALITY (and probably Lorentz symmetry **OFF-SHELL** (Greenberg) 2002.)
- Current ExperimentaL Situation: WMAP + all other data on neutrinos, including KamLAND: Simplest model of CPTV with Dirac neutrino masses is **marginally ruled out**?
- But this is **only one way of CPTV**. Remember departure from Quantum Mechanical Evolution (QMV) (as in SOME QG models) QMV-CPTV may induce neutrino oscillations and CPTV not related to masses (Fogli, Lisi, Marrone 2000).
- stringent limits on CPTV-ν by $\beta\beta$-decay if true (Klapdor-Kleingrothaus, Paes and Sarkar 2000)

NEUTRINO ANOMALIES & CPTV

Interpreting the LSND anomaly as CPT-violation in the neutrino sector

LSND anomaly : observed oscillations in $\bar{\nu}_\mu - \bar{\nu}_e$

(initial 2.6σ hint for $\nu_\mu - \nu_e$ decreased to 0.6σ)

Proposal of Murayama and Yanagida (hep-ph/0010178) (see also: Barenboim + Lykken, Barenboim *et al.* hep-ph/0108199):

CPT-Violating (CPTV) (Dirac) Mass spectrum* : $m_\nu \neq m_{\bar{\nu}}$.

Expts like MiniBooNE (looks for $\nu_\mu \to \nu_e$) can directly test such hypotheses in the immediate future.

If CPTV then signals in atmospheric & solar ν oscillations.

Recent results from KamLAND (hep-ex/0212021) + ATMOSPHERIC DATA

$\to$ disfavour marginally CPTV-Dirac mass scenario (A. Strumia, hep-ph/0201134 v4 (April 03), Addendum).

CPTV-ν-mass scenario and LSND anomaly

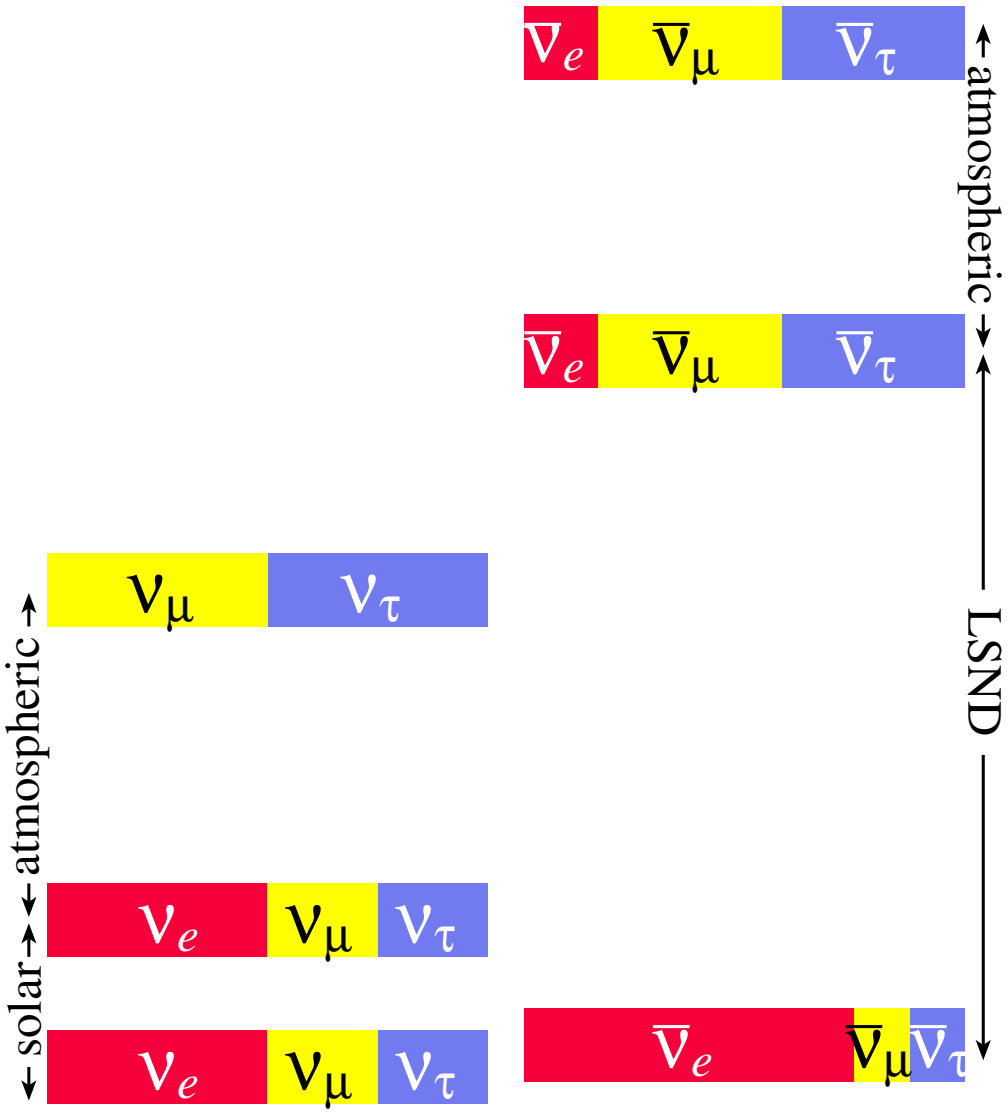


Figure 5: The CPTV spectrum proposed by Murayama-Yanagida. To explain LSND we need $m_\nu^2 - m_{\bar{\nu}}^2 \sim 0.1\ \mathrm{ev}^{-2} = 10^{-19}\ \mathrm{GeV}^2$.

* ν CPTV (Dirac) Mass Spectrum

(Barenboim, Borisov, Lykken, Smirnov hep-ph/0108199)

Relax Locality (but maintain Lorentz Invariance), hence CPTV natural.

Dirac Mass: $m_i = \tan\beta \overline{m}_i$, $i = 1, 2, 3$ (three neutrino flavours) $\tan\beta = 0$ or $\cot\beta = 0$ MAXIMAL CPTV. $\tan\beta = 1$ CPT conserved.

Explain LSND without Sterile ν.

why CPTV only in ν? Brane world scenaria (ν can propagate in the bulk like gravity ?)

No knowledge of mixing matrix

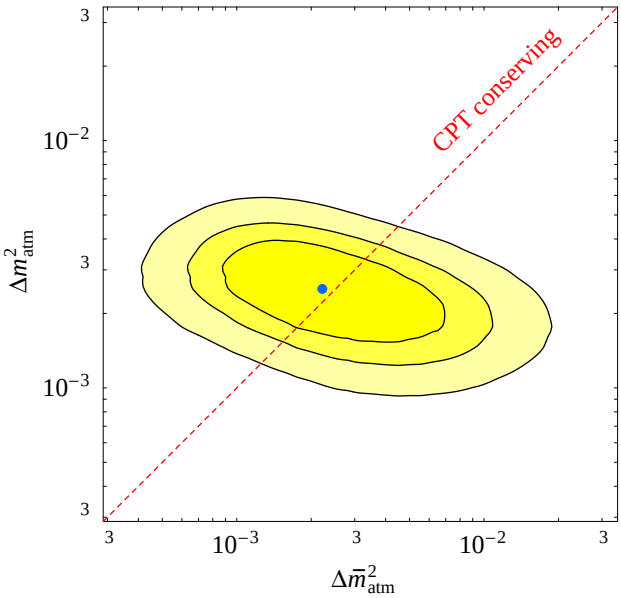
Specific Model (BARENBOIM, LYKKEN hep-ph/0210041): "Homeotic ν" (like Dirac theory both +ve,-ve energies):

$$\psi_+(x) = u_+(p)e^{-ip\cdot x}, \quad p^2 = m^2, \; p_0 > 0$$
$$\psi_-(x) = u_-(p)e^{-ip\cdot x}, \quad p^2 = m^2, \; p_0 < 0$$
$$(p_\mu \gamma^\mu - m\epsilon(p_0))u_\pm(p) = 0, \; \epsilon(p_0) = \text{sign function}$$
$$S = \int d^4x \bar{\psi} i\partial_\mu \gamma^\mu \psi + \frac{im}{2\pi}\int d^3x dt dt' \bar{\psi}(t)\frac{1}{t-t'}\psi(t')$$

NB: Lorentz invariance maintained ON SHELL only. $(\epsilon(p_0))$ but Locality relaxed. OFF-SHELL LV (Greenberg 2002)

Fit of SK & K2K Data

(A. Strumia, hep-ph/0201134 v4, addendum.)



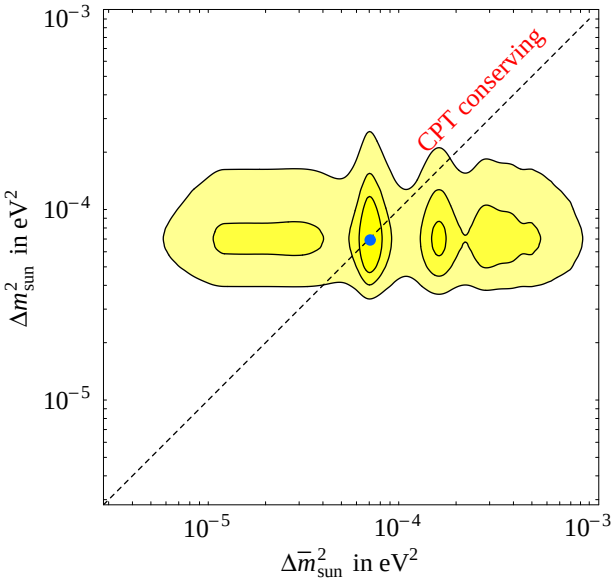


Figure 6: Atmospheric $m_\nu - m_{\overline{\nu}}$ (68, 90, 99 %, 2 d.o.f.)

Figure 7: For solar & reactor data (68, 90, 99 %, 2 d.o.f.)

CPTV mass scenario marginally disfavoured?

WMAP Data and Neutrinos

Cosmology (WMAP): $\sum m_\nu < 0.69$ eV (95% CL, 1 d.o.f.)
(one can include sterile ν)

$$P_{\nu_e \to \nu_e} = 1 - S\sin^2\theta_{es}, \quad P_{\nu_\mu \to \nu_\mu} = 1 - S\sin^2\theta_{\mu s},$$
$$P_{\nu_e \to \nu_\mu} = S\sin^2\theta_{LSND} \; (\sin^2\theta_{LSND} \simeq \tfrac{1}{4}\sin^2 2\theta_{es}\sin^2 2\theta_{\mu s})$$

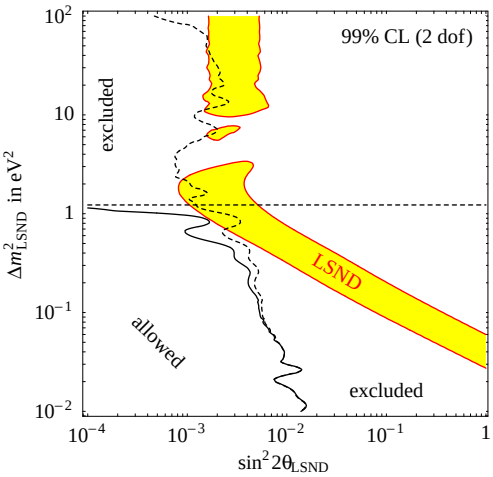


Figure 8: Upper half plane disfavoured by WMAP. Dashed curved line: upper bound from all other ν expts.

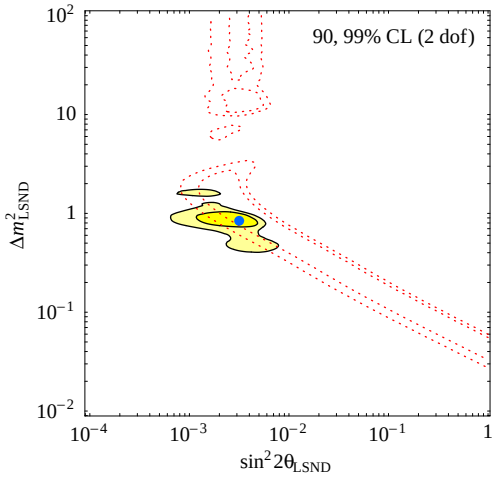


Figure 9: Best fit , all data; 3 + 1 sterile ν solution (CPT cons.) disfavoured by WMAP since $(\Delta m^2_{LSND})^{1/2} \simeq \sum m_\nu$.

(A. Strumia, hep-ph/0201134 v4, addendum.)

Data Summary and Interpretations

model & no. of free parameters		$\Delta\chi^2$	mainly incompatible with	main future test
ideal fit (no known model)		0		?
$\Delta L = 2$ decay $\bar{\mu} \to \bar{e}\bar{\nu}_\mu\bar{\nu}_e$	6	12	KARMEN	TWIST
$3+1:\ \Delta m^2_{\rm sterile} = \Delta m^2_{\rm LSND}$	9	$6+9$?	BUGEY + cosmology?	MINIBOONE
3 ν and CPTV (no $\Delta\bar{m}^2_{\rm sun}$)	10	15	KamLAND	KAMLAND
3 ν and CPTV (no $\Delta\bar{m}^2_{\rm atm}$)	10	25	SK atmospheric	$\bar{\nu}_\mu$ LBL?
normal 3 neutrinos	5	25	LSND	MINIBOONE
$2+2:\ \Delta m^2_{\rm sterile} = \Delta m^2_{\rm sun}$	9	30	SNO	SNO
$2+2:\ \Delta m^2_{\rm sterile} = \Delta m^2_{\rm atm}$	9	50	SK atmospheric	ν_μ LBL

Table 1: Interpretations of solar, atmospheric and LSND data, ordered according to the quality of their global fit. A $\Delta\chi^2 = n^2$ roughly signals an incompatibility at n standard deviations. (A. Strumia, hep-ex/0304039)

What about four $(3+1$ or $2+2)$ ν and CPTV ?

Four ν + CPTV

(Barger, Marfatia & Whisnant 2003)

CPTV $\rightarrow$ different flavor mixing between ν, $\bar{\nu}$:

$\nu_a = \sum_{i=1}^{4} U_{ai}^{*} \nu_i, \qquad \bar{\nu}_a = \sum_{i=1}^{4} \bar{U}_{ai} \bar{\nu}_i,$

with $U \neq \bar{U}$ due to CPTV.

- 3 + 1 models: one ν mass well separated from others, sterile ν couples only to isolated state.

  Oscillation probs: $P_{\nu_i \rightarrow \nu_i}(|U_{ij}|^2) \neq P_{\bar{\nu}_i \rightarrow \bar{\nu}_i}(|\bar{U}_{ij}|^2)$

  Bugey binds $|\bar{U}_{e4}|$ and CDHSW binds $U_{\mu 4}$ but no tight constraints for $|\bar{U}_{\mu 4}|$, U_{e4}. (Contrast with (3 + 1)ν CPT conserving models where $U = \bar{U}$.).

  Hence (3 + 1)ν + CPTV still viable.

- 2 + 2 models: sterile ν couples to solar and atmospheric ν oscillations. This structure is only permitted in $\bar{\nu}$ sector. Even with CPTV 2+2 models strongly disfavoured by data.

3+1 ν + CPTV

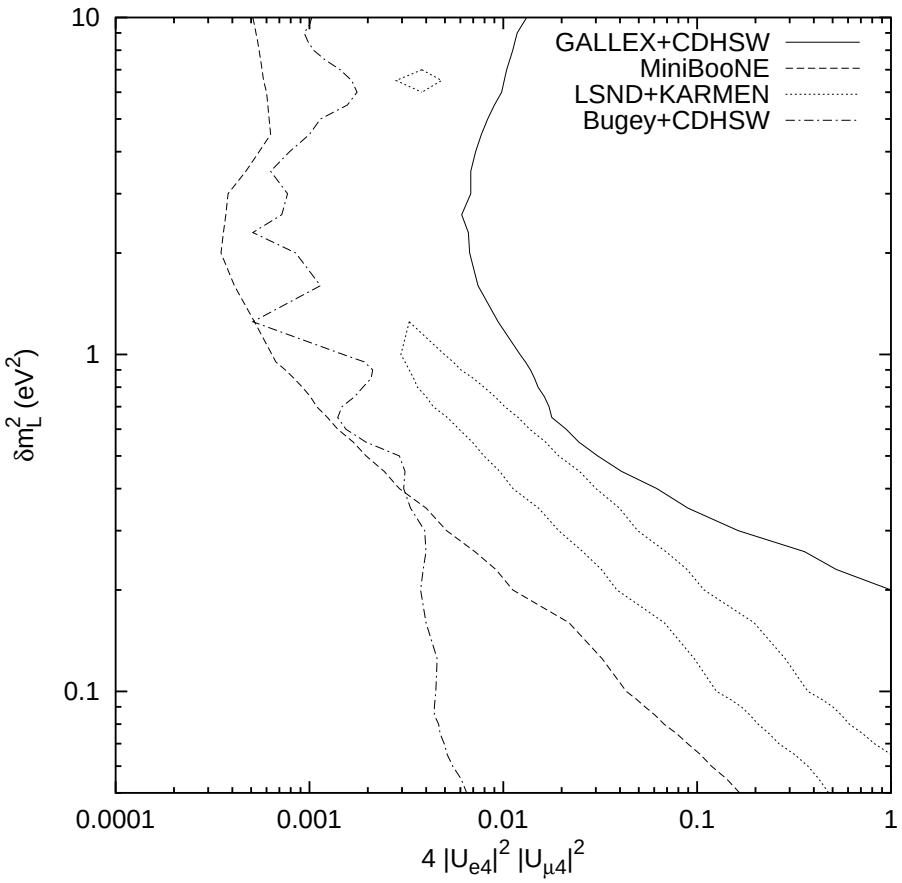


Figure 10: Upper bound (solid) on the $\nu_\mu \to \nu_e$ oscillation amplitude $4|U_{e4}|^2|U_{\mu 4}|^2$ from the GALLEX limit on $|U_{e4}|$ and the CDHSW limit on $|U_{\mu 4}|$ (90% C. L. results are used in both cases). The dot-dashed line is the 99% C. L. upper bound from Bugey and CDHSW if CPT is conserved. Also shown are the expected sensitivity (dashed) of the MiniBooNE experiment and, for comparison, the allowed region (within the dotted lines) for $4|\bar{U}_{e4}|^2|\bar{U}_{\mu 4}|^2$ from a combined analysis of LSND and KARMEN data, both at the 90% C. L .

3+1 $\bar{\nu}$ + CPTV

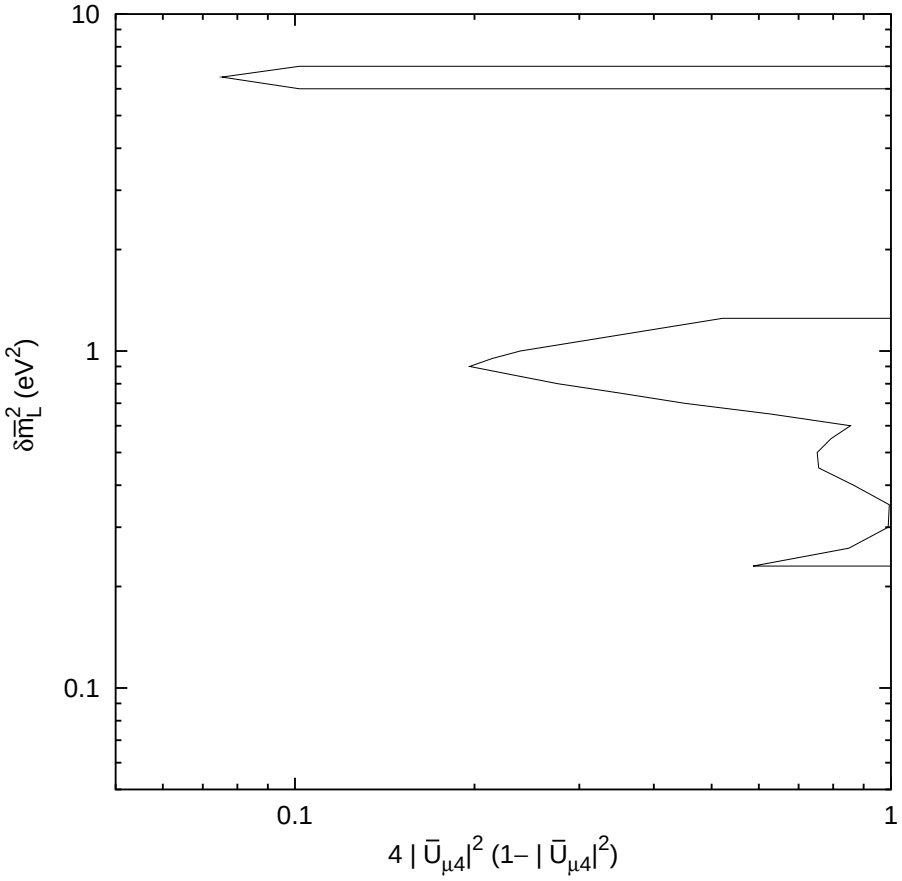


Figure 11: Lower bounds on $4|\bar{U}_{\mu 4}|^2(1-|\bar{U}_{\mu 4}|^2)$ (the amplitude for atmospheric $\bar{\nu}_\mu$ survival at the LSND mass scale) from the Bugey limit on $\bar{\nu}_e$ disappearance and the $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillation amplitude indicated by LSND and KARMEN (90% C. L. results are used in both cases).

2+2 ν + CPTV

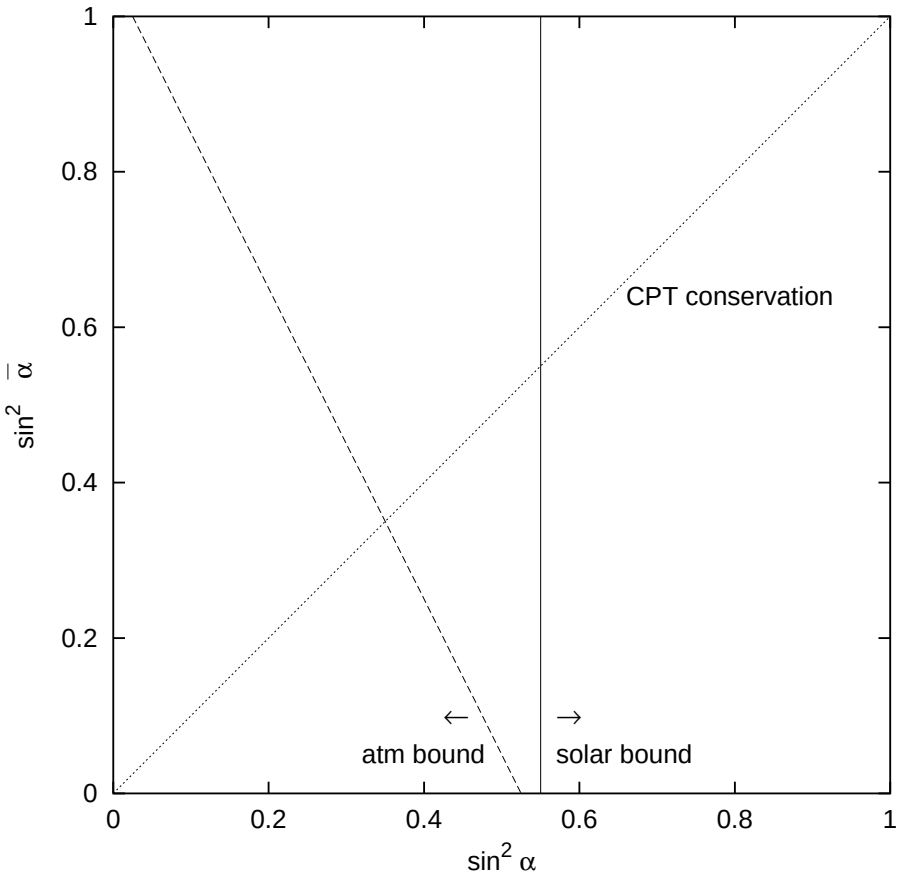


Figure 12: Constraints on sterile neutrino mixing angles α and $\bar{\alpha}$ from solar (solid) and atmospheric (dashed) data. The dotted line is the prediction if CPT is conserved.

CONCLUSIONS

Neutrino Physics may provide a very useful guide in our quest for a theory of Quantum Gravity, in particular stringent constraints on CPT Violation.

Neutrino oscillation experiments provide stringent bounds on many quantum gravity models, entailing Lorentz Invariance Violation. There are also plenty of low energy nuclear and atomic physics experiments which yield stringent bounds in models with Lorentz (LV) and CPT violation. Frame dependence of LV effects crucial.

But Quantum Gravity may exhibit Lorentz Invariant (and hence frame independent) CPTV Decoherence. Theoretically the presence of an environment may be consistent with Lorentz Invariance (Millburn 2003). This scenario is still compatible with all the existing ν data, given that the parameters of such models are highly model dependent, and thus subject to constraints by experiment.

More work (Theory & Expt) to be done before conclusions are reached...