

NO-VE, "Neutrino Oscillations in Venice", December 3-5 2003

Leptogenesis and Neutrino Mixing Data

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Reference paper, with W.Buchmüller and M. Plümacher,
to appear soon on hep-ph

Baryon asymmetry of the Universe

- CMB + cosmic rays exclude a baryon symmetric universe with matter- anti matter domains, on scales as large as the whole horizon
(Cohen, De Rujula, Glashow, '98)
- from CMB acoustic peaks (WMAP) + large scale structure (SLOAN):

$$\eta_B^{CMB} = (6.3 \pm 0.3) \times 10^{-10}$$

(Tegmark et al. 2003)

- ... in very good agreement with the determination from SBBN + primordial Deuterium measurements :

$$\eta_B^{SBBN} = (6.1 \pm 0.5) \times 10^{-10}$$

(Cyburt et al. 2001, Kirkman et al. 2003)

Models of Baryogenesis

- at the Planck scale
 - from phase transitions
 - Electroweak Baryogenesis
 - * in the Standard Model
 - * in the MSSM
 - * ...
 - ...
 - Affleck-Dine
 - at preheating
 - Q-Balls
 - from black holes evaporation
 - spontaneous baryogenesis
 - ...
 - from heavy particle decays
 - GUT baryogenesis
 - leptogenesis

Heavy particle decays

1. Kolb & Turner toy model

$$X \left[\begin{array}{ccc} \xrightarrow{\Gamma} & b \ b \\ \xrightarrow{\bar{\Gamma}} & \bar{b} \ \bar{b} \end{array} \right] (\Delta_{B-L} = +1)$$

$$\beta = 2, B_b = 1/2 \Rightarrow |\Delta B| = 1$$

2. Leptogenesis

$$N_i \left[\begin{array}{ccc} \xrightarrow{\Gamma} & l \ \bar{\Phi} \\ \xrightarrow{\bar{\Gamma}} & \bar{l} \ \Phi \end{array} \right] (\Delta_{B-L} = -1)$$

$$\beta = 1, L_l = 1 \Rightarrow |\Delta L| = 1$$

After sphaleron conversion:

$$N_B^f \simeq \frac{1}{3} N_{B-L}^f, \quad N_L^f \simeq -\frac{2}{3} N_{B-L}^f$$

- CP asymmetry parameter:

$$\varepsilon = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$

- Total decay ~~parameter rate~~:

$$\Gamma_D = \Gamma + \bar{\Gamma} = \Gamma_D^{\text{rest}} \left\langle \frac{1}{\gamma} \right\rangle$$

Out of equilibrium decays

$$z = \frac{M_X}{T}, \quad D = \frac{\Gamma_D}{H z}, \quad \tilde{\varepsilon} = \varepsilon \Delta_{B-L}$$

$$N_i = n_i R^3, \quad N_\gamma(z \ll 1) = 1$$

$$\frac{dN_X}{dz} = -D(z) N_X(z)$$

$$\frac{dN_{B-L}}{dz} = -\tilde{\varepsilon} \frac{dN_X}{dz}$$

$$N_{B-L}(z) = N_{B-L}^i + \tilde{\varepsilon} [N_X^i - N_X(z)]$$

$$N_X(z) = N_X^i e^{-\int_{z_i}^z dz' D(z')}$$

$$N_{B-L}^f = N_{B-L}^i + \tilde{\varepsilon} N_X^i$$

$$\eta_B^f \simeq \frac{1}{3} \frac{N_{B-L}^f}{d}$$

$$(d = N_\gamma^{\text{rec}}/N_\gamma^{\text{B}} \sim 30)$$

- Efficiency factor:

$$N_{B-L}(z) = N_{B-L}^i + \tilde{\varepsilon} \frac{N_X^i}{N_{X0}^{\text{eq}}} \kappa(z)$$

$$\kappa_f = \frac{N_X^i}{N_{X0}^{\text{eq}}}$$

$$(N_{N_1 0}^{\text{eq}} = 3/4)$$

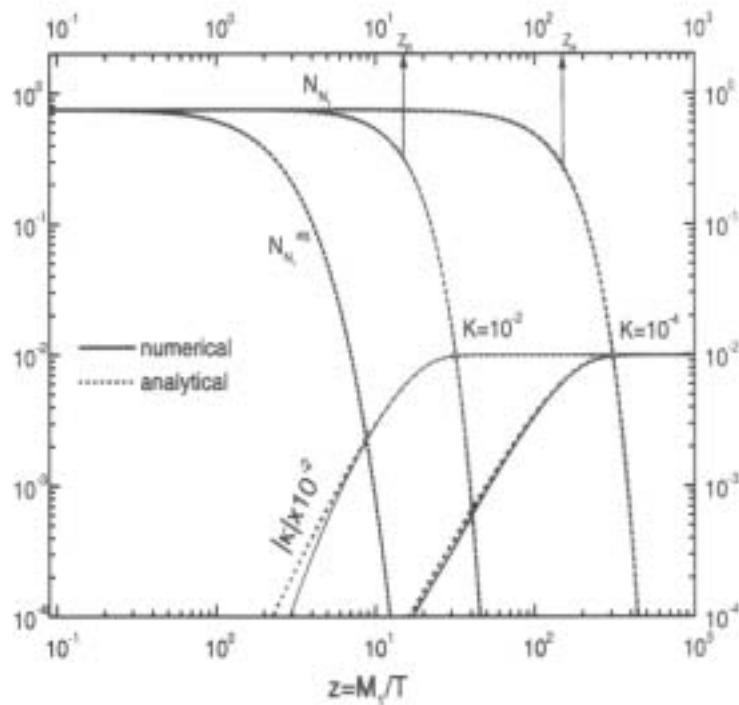
- Decay parameter:

$$K = \frac{\Gamma_D^{\text{rest}}}{H|_{z=1}} \Rightarrow D = K \left\langle \frac{1}{\gamma} \right\rangle z$$

- Dilation factor:

$$\left\langle \frac{1}{\gamma} \right\rangle = \frac{K_1(z)}{K_2(z)} \simeq \frac{z}{z + \frac{15}{8}}$$

in the case of leptogenesis ($X = N_1$) and $N_X^i = N_{N_1}^{\text{th}}$:



- Decay Temperature ($T_d = M_1/z_d$):

$$\tau_X = t_U(z = z_d) \Rightarrow z_d \simeq \sqrt{\frac{2}{K}}, \quad (K \ll 1)$$

For $K \gtrsim 1$ inverse decays have to be taken into account

Decays and Inverse Decays

$$\begin{aligned}\frac{dN_X}{dz} &= -D N_X + D N_X^{\text{eq}} \\ \frac{dN_{B-L}}{dz} &= -\tilde{\varepsilon} \frac{dN_X}{dz} - W_{ID} N_{B-L}\end{aligned}$$

$$\tilde{\varepsilon} = \frac{N_X^{\text{eq}}}{N_{b,i}^{\text{eq}}} = \frac{\beta}{4} K z^3 K_1(z)$$

$$N_{B-L}(z) = N_{B-L}^{\text{in}} e^{-\int_{z_{\text{in}}}^z dz' W_{ID}(z')} + N_{X0}^{\text{eq}} \tilde{\varepsilon} \kappa(z)$$

$$\kappa(z; \dots, z_{\text{in}}) = -\frac{1}{N_{X0}^{\text{eq}}} \int_{z_{\text{in}}}^z dz' \left[\frac{dN_{N_1}}{dz'} \right] e^{-\int_{z'}^z dz'' W_{ID}(z'')}$$

Two regimes:

- Weak wash-out regime for $K \lesssim 1$

(the out of equilibrium decays picture has to be recovered asymptotically in the limit $K \rightarrow 0$)

- Strong wash-out regime for $K \gtrsim 1$

Strong wash-out regime

initial thermal abundance: $N_X^{\text{in}} = N_{X0}^{\text{eq}}$

$$K \gtrsim 1 \Rightarrow \boxed{\frac{dN_X}{dz} \simeq \frac{dN_X^{\text{eq}}}{dz}} = -\frac{1}{2} N_{X0}^{\text{eq}} z^2 K_1(z)$$

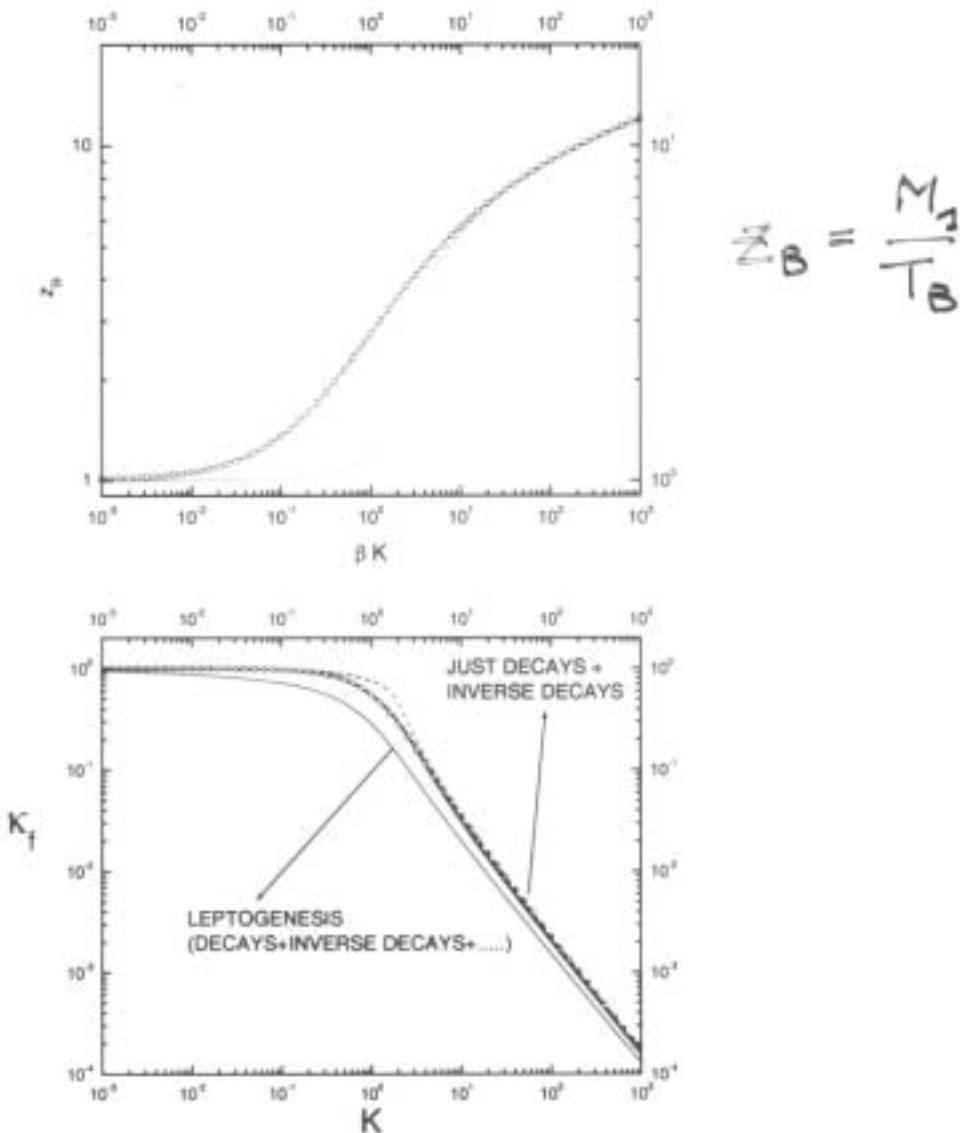
$$\begin{aligned} \kappa_f &= \frac{1}{2} \int_0^\infty dz' z'^2 K_1(z') e^{-\frac{\beta K}{4} \int_{z'}^\infty dz'' z''^2 K_1(z'')} \\ &= \int_0^\infty dz' e^{-\psi(z', z)} \quad (\text{Laplace integral}) \\ &\simeq \int_{z_B - \Delta z_B}^{z_B + \Delta z_B} dz' e^{-\psi(z', z)} \quad \left(\frac{d\psi}{dz'} \Big|_{z=z_B} = 0 \right) \\ &\simeq \frac{1}{2} \int_0^\infty dz' z'^2 K_1(z') e^{-\frac{\beta K z_B}{4} \int_{z'}^\infty dz'' z''^2 K_1(z'')} \\ &\boxed{\kappa_f \simeq \frac{2}{\beta K z_B} \left(1 - e^{-\frac{\beta K z_B}{2}} \right)} \end{aligned}$$

This expression extends the KT result, $\kappa_f = 1/(K z_B)$, that is obtained for $\beta = 2$ and $K \gg 1$ ^a.

^aBut the correct definition $K = \Gamma_D^{\text{rest}}/H|_{z=1}$ has to be used instead of $K_{KT} = (\Gamma_D/[2H])_{z=1} \simeq K/6$.

The quantity z_B is found solving $(d\psi/dz')|_{z_B} = 0$

$$\Rightarrow \frac{K}{4} z_B^3 e^{-z_B} \sqrt{1 + \frac{\pi}{2} z_B} = z_B - 1$$



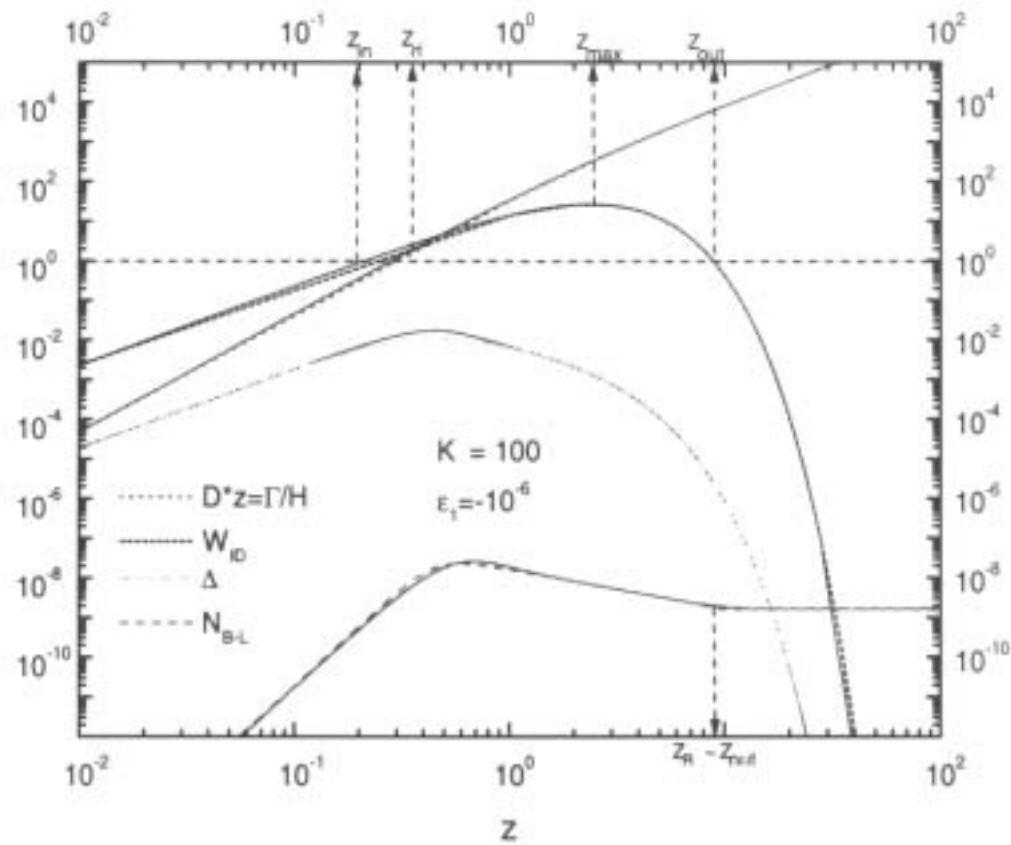
Using the fit: $z_B(\beta K) \simeq 1 + 2.4 \ln(1 + \beta K)^{0.8} \Rightarrow$

- agreement also for $K \simeq 1$
- explicit analytical expression for κ_f .

Evolution of the asymmetry

Let us define:

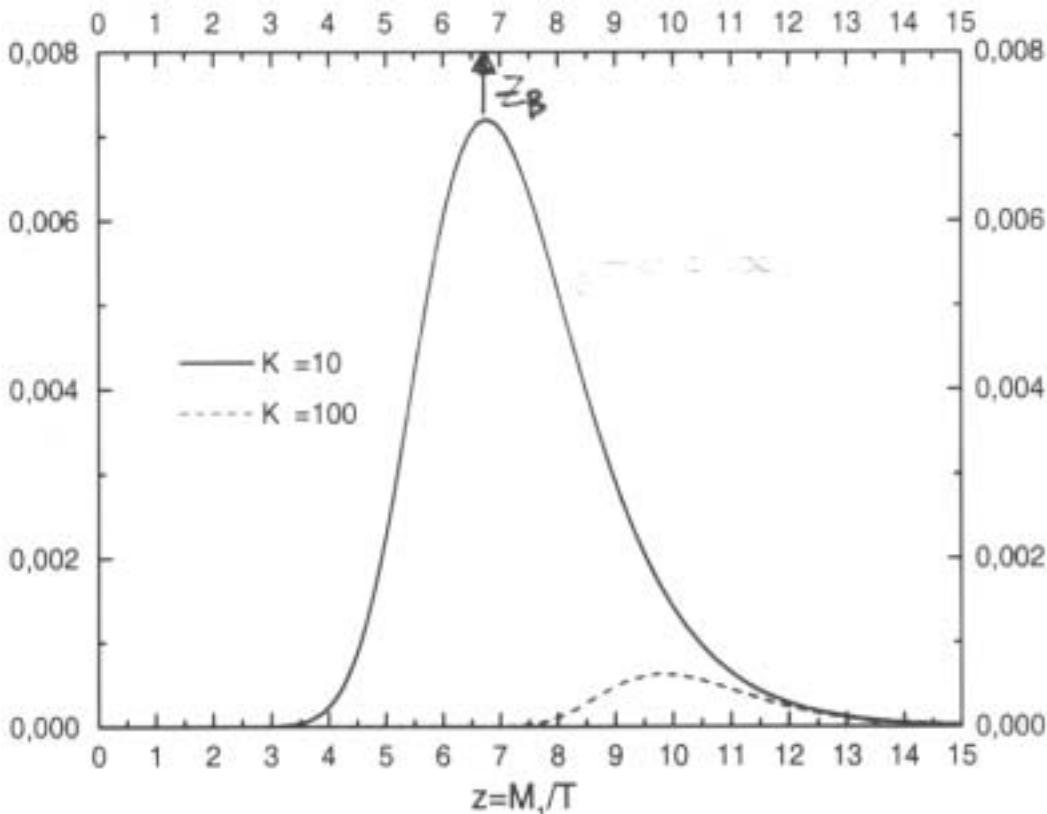
$$\Delta(z) = N_{N_L}(z) - N_{N_R}(z)$$



Temperature of baryogenesis

$$\eta_B \simeq 10^{-2} \tilde{z} \kappa_f$$

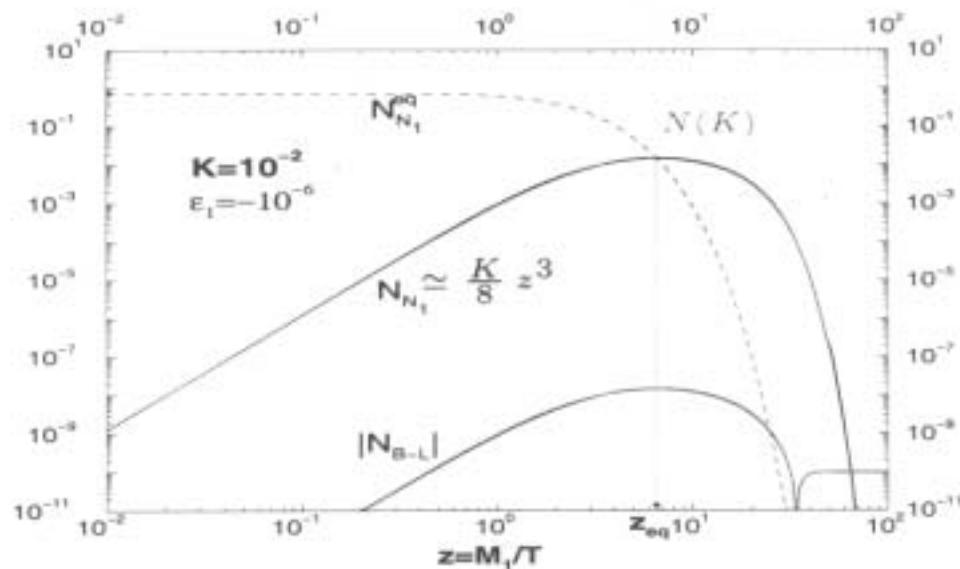
$$\kappa_f = \int_0^\infty dz' e^{-\psi(z', \infty)}$$



- $T_B = M_1/z_B$ can be well regarded as the temperature of baryogenesis
- The stage for $T \gtrsim T_B$ does not affect the final prediction
⇒ many interesting consequences

Neutrino production

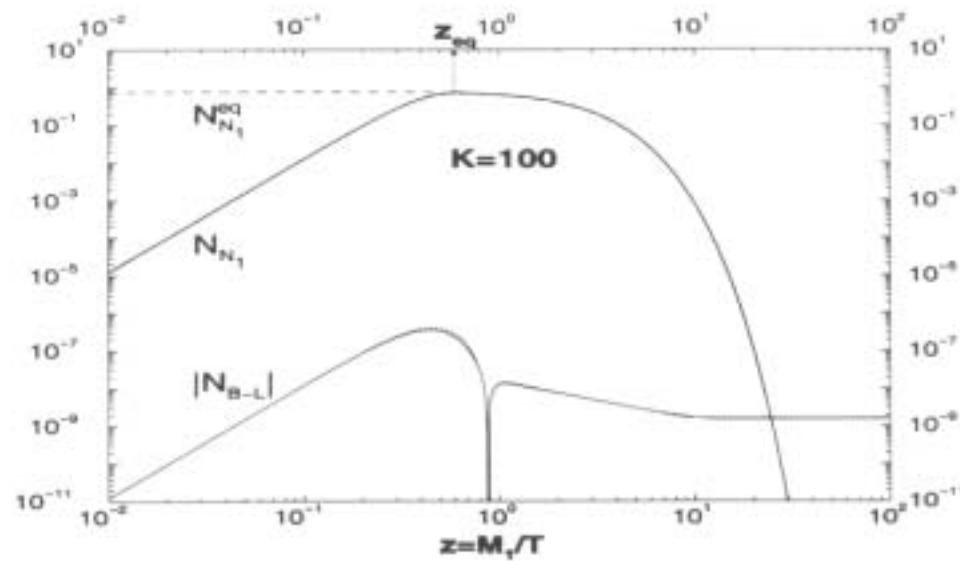
weak wash-out regime



$$\kappa = -|\kappa^-| + \kappa^+ \Leftrightarrow N_{B-L} = -|N_{B-L}^-| + N_{B-L}^+$$

$$\frac{1}{N(K)} \simeq \frac{16}{9\pi K} + \frac{4}{3}$$

strong wash-out regime



Final efficiency factor: summary

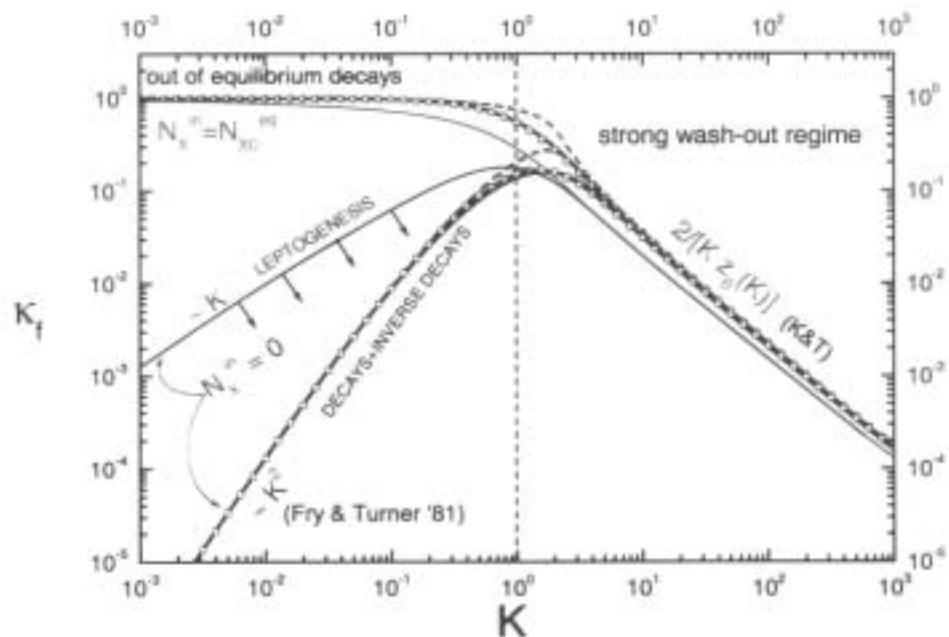
- Initial thermal abundance ($\beta = 1$)

$$\kappa_f \simeq \frac{2}{K z_B} \left(1 - e^{-\frac{K z_B}{2}} \right)$$

- Neutrino production ($\beta = 1$)

$$\kappa_{\tilde{\nu}}^+ = -2 e^{-\frac{3\pi}{8} K} \left[e^{\frac{2}{3} K N(K)} - 1 \right]$$

$$\kappa_{\tilde{\nu}}^- = \frac{2}{K z_B} \left[1 - e^{-\frac{2}{3} K z_B N(K)} \right]$$



Leptogenesis . . .

(Fukugita, Yanagida '86)

- . . . is the cosmological consequence of (minimal) seesaw:

$$m_\nu = -m_D \frac{1}{M} m_D^T$$

- three new heavy RH neutrinos: N_1 , N_2 and N_3 with masses $M_{\text{ew}} \ll M_1 \leq M_2 \leq M_3$;
- decaying particles $X = N_1$;
- effective neutrino mass

$$\tilde{m}_1 \equiv \frac{(m_D^\dagger m_D)_{11}}{M_1}$$

- total decay rate

$$\Gamma_D^{\text{rest}} = \frac{1}{8\pi v^2} \tilde{m}_1 M_1^2$$

- decay parameter

$$K = \frac{\Gamma_D^{\text{rest}}}{H|_{z=1}} = \frac{\tilde{m}_1}{m_\star}$$

- equilibrium neutrino mass

$$m_\star = \frac{v^2}{M_\star} \simeq 10^{-3} \text{ eV}$$

$$M_\star = \frac{3\sqrt{5}}{16\pi^{5/2}} \frac{M_{Pl}}{\sqrt{g_\star}}$$

$$M_* \simeq 3 \times 10^{16} \text{ GeV} !$$

CP asymmetry

- Interference between tree level and (vertex + self energy) one-loop diagrams

$$\Rightarrow \varepsilon_1 \simeq \frac{1}{8\pi v^2 (m_D m_D^\dagger)_{11}} \sum_{i=2,3} \text{Im} \left[(m_D m_D^\dagger)_{i1}^2 \right] \times \\ \times \left[f_V \left(\frac{M_i^2}{M_1^2} \right) + f_S \left(\frac{M_i^2}{M_1^2} \right) \right]$$

(Flanz, Paschos, Sarkar'95; Covi, Roulet, Vissani'96; Buchmüller, Plümacher'98)

- Maximum CP asymmetry:

$$\varepsilon_1 = \varepsilon_1^{\text{max}} \sin \delta_L$$

$$\varepsilon_1^{\text{max}} = \frac{3}{16\pi} \frac{M_1 m_3}{v^2} \left(1 - \frac{m_1}{m_3} \sqrt{1 + \frac{m_3^2 - m_1^2}{\tilde{m}_1^2}} \right)$$

(Asaka et al.'99; Goldberg '00; Barbieri et al.'00; Hamaguchi, Murayama, Yanagida '01; Davidson, Ibarra '02; Buchmüller, PDB, Plümacher '03)

Range of \tilde{m}_1

- A useful tool is given by (Casas, Ibarra '01):

$$\Omega = D_m^{-\frac{1}{2}} U^\dagger m_D D_M^{-\frac{1}{2}} \Rightarrow \Omega^T \Omega = I$$

where $U^\dagger m_\nu U^* = -\text{diag}(m_1, m_2, m_3) \equiv -D_m$

- with this orthogonal matrix one can write

(Fujii, Hamaguchi, Yanagida '02):

$$\tilde{m}_1 = \frac{(m_D^\dagger m_D)_{11}}{M_1} = \sum_{j=1}^3 m_j |\Omega_{j1}|^2 \geq m_i$$

This is the only completely model independent information on \tilde{m}_1 (meaningful only if $m_1 \neq 0$)

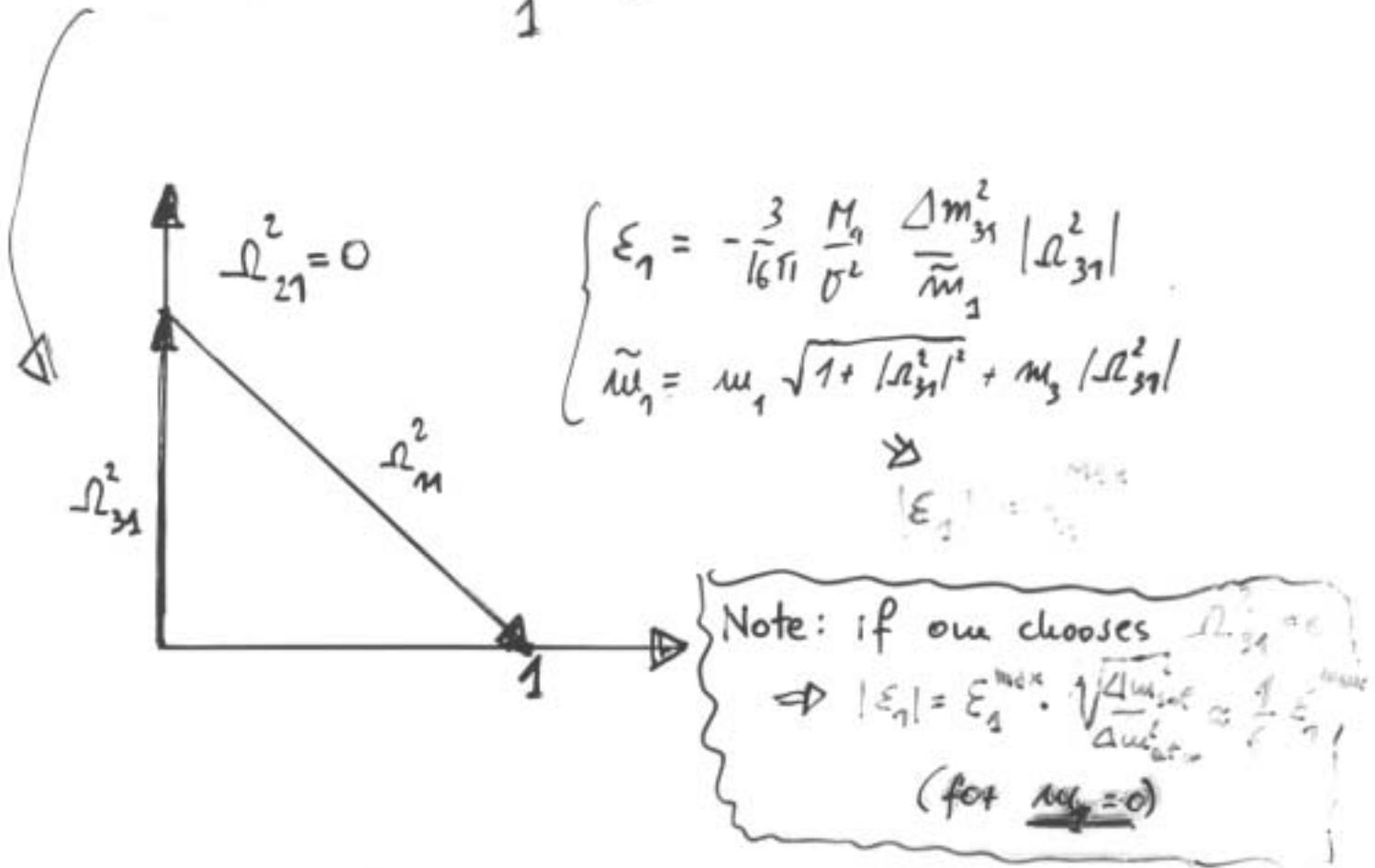
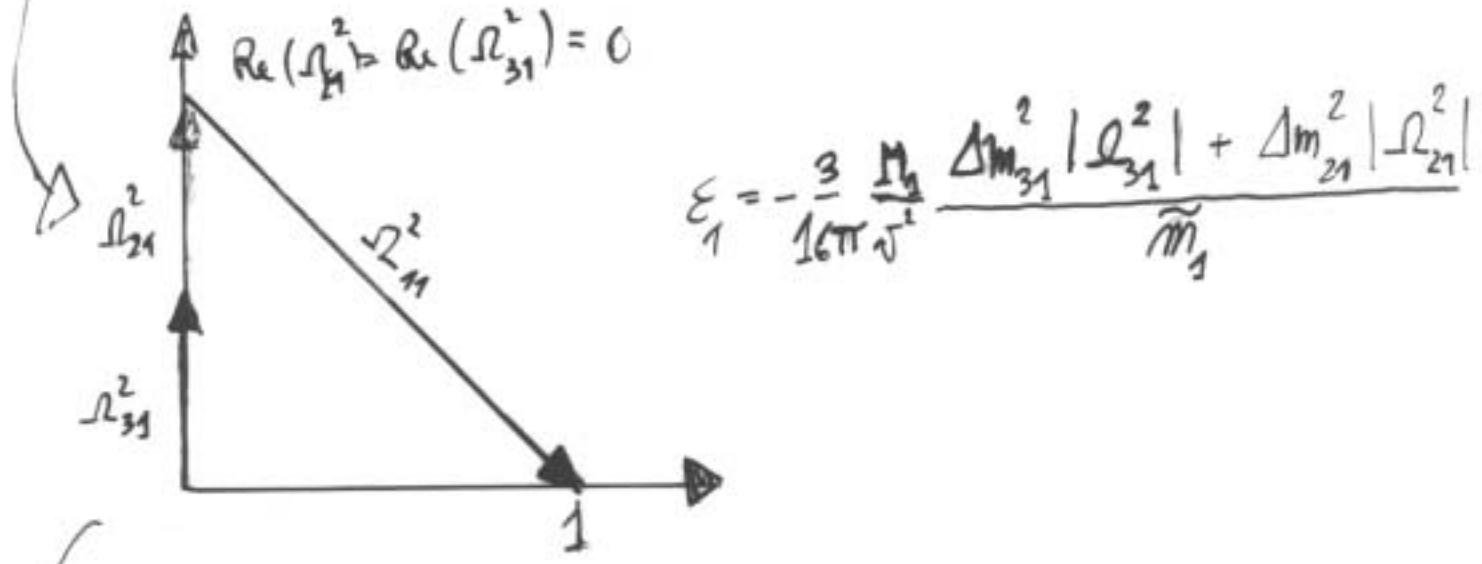
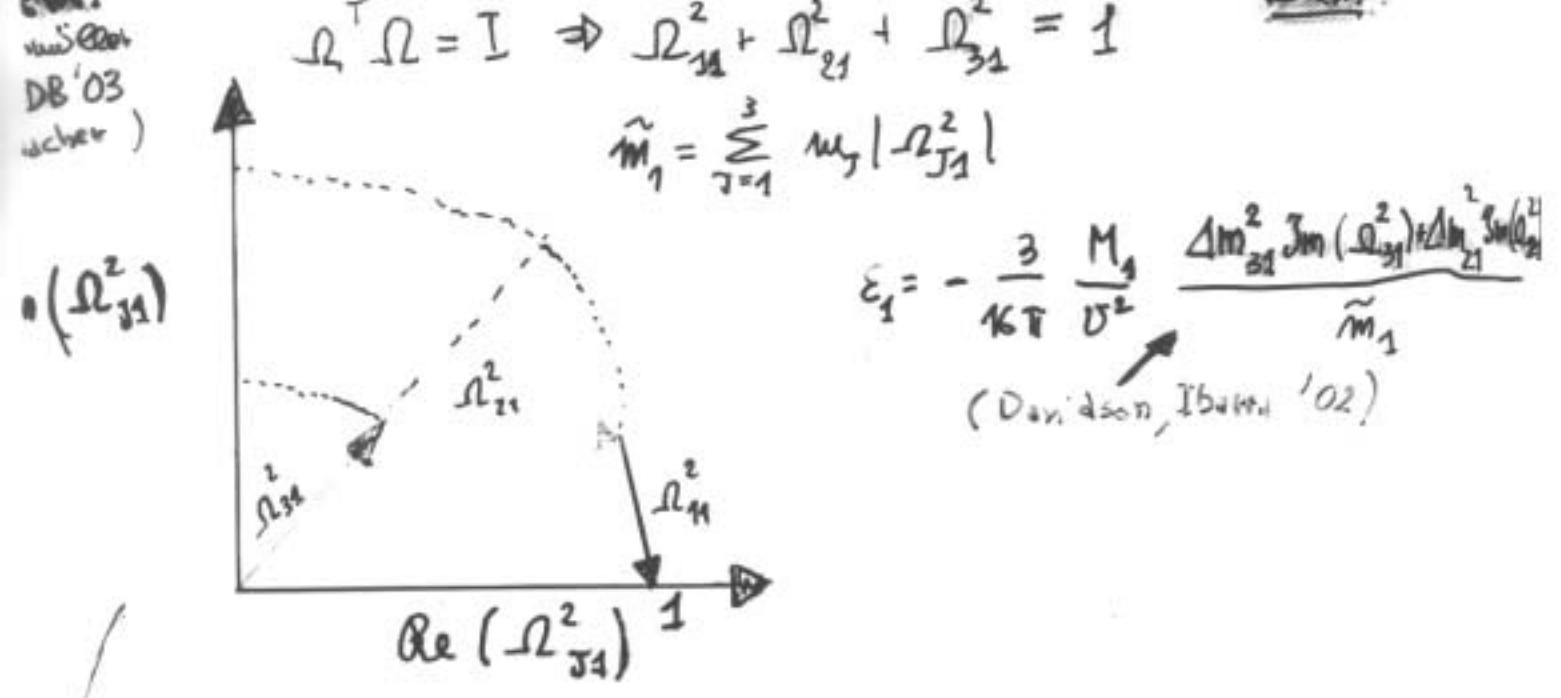
- barring strong cancellations then:

$$\tilde{m}_1 \leq m_3 \sum_i |\Omega_{j1}|^2 \approx m_2 \left| \sum_i \Omega_{j1}^2 \right| = m_3$$

- for quasi-degenerate neutrinos $\tilde{m}_1 \simeq m_i \gtrsim 0.1 \text{ eV}$;
- for hierarchical neutrinos ($m_1 \ll m_{\text{sol}}$) and for $\sin \delta_L \sim 1$ ‘typically’:

$$10^{-2} \text{ eV} = \mathcal{O}(m_{\text{sol}}) < \tilde{m}_1 < \mathcal{O}(m_{\text{atm}}) = 10^{-1} \text{ eV}$$

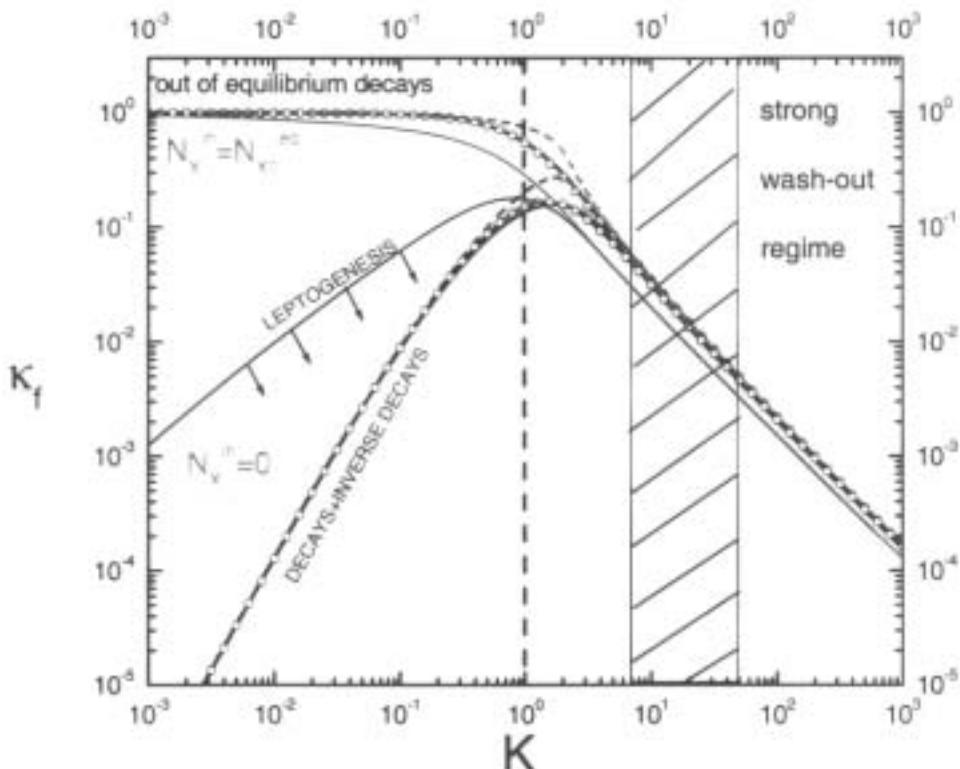
$$(m_{\text{atm,sol}} \equiv \sqrt{\Delta m_{\text{atm,LMA}}^2})$$



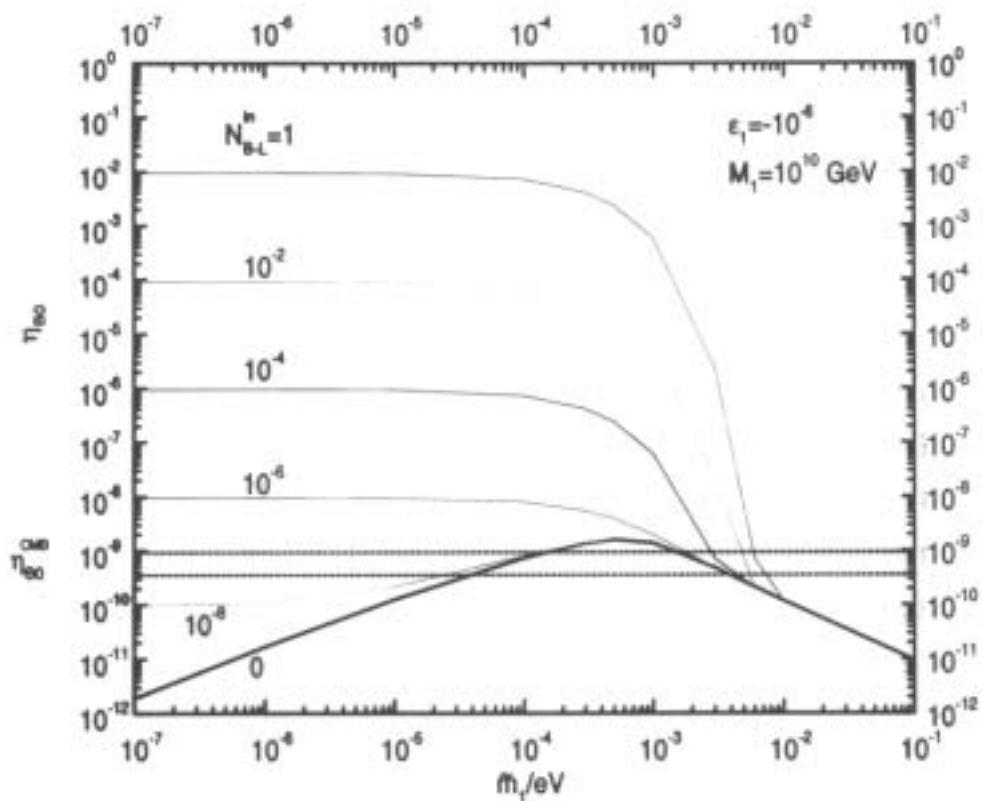
Leptogenesis K range

Translating \tilde{m}_1 in terms of K :

$$7 \simeq K_{\text{sol}} \lesssim K \lesssim K_{\text{atm}} \simeq 50$$

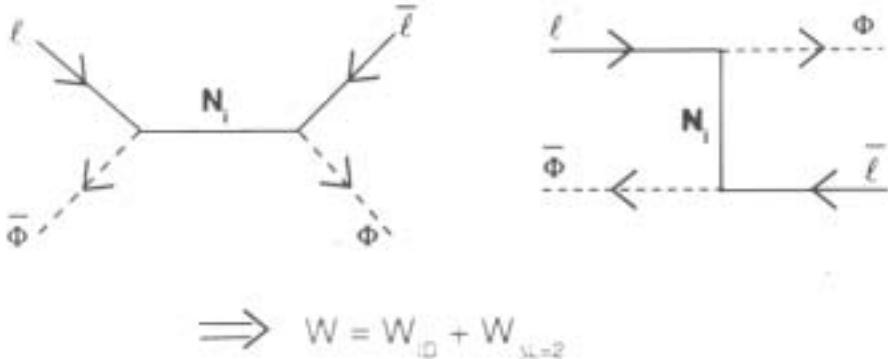


Dependence on the initial asymmetry

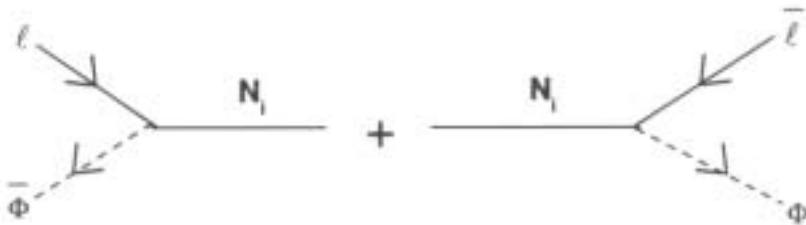


For $\tilde{m}_1 \gg 5 \times 10^{-4} \text{ eV}$ and $T_{\text{in}} \gtrsim M_1$ the final baryon asymmetry is independent on the initial conditions

$\Delta L = 2$ processes



One has to be careful not to double count the corresponding on-shell processes:



- It is important that on shell contributions are properly subtracted
- It has been noticed that (Buchmuller, PDB, Plumacher, '02):

$$W_{\Delta L=2}(z) = W_{\Delta L=2}^{\text{res}}(z) + \Delta W(z)$$

$$W_{\Delta L=2}^{\text{res}} = \frac{1}{2} W_{ID} \propto \bar{m}_1 \quad !!$$

$$\Delta W(z) \propto \frac{M_1 \sum_i m_{\nu_i}^2}{z^2}$$

- ΔW is important for $M_1 \gtrsim 10^{13} \text{ GeV} (0.1 \text{ eV} / \sum_i m_{\nu_i}^2 / \text{eV})^2$
- The first term is suspicious ... and indeed recently the subtr. procedure has been revisited and this term has been found to be spurious (Giudice et al '03)
- What has been changed? This term was equivalent to take $\beta = 3/2$ instead of 1 mainly affecting the strong wash out regime:

$$\kappa_f = 4/[3 K z_B(3K/2)] \longrightarrow 2/[K z_B(K)]$$

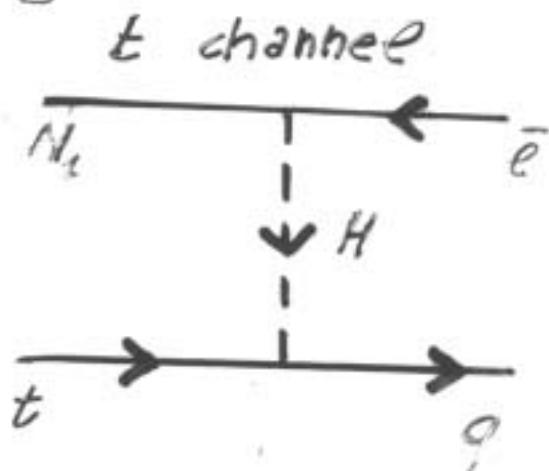
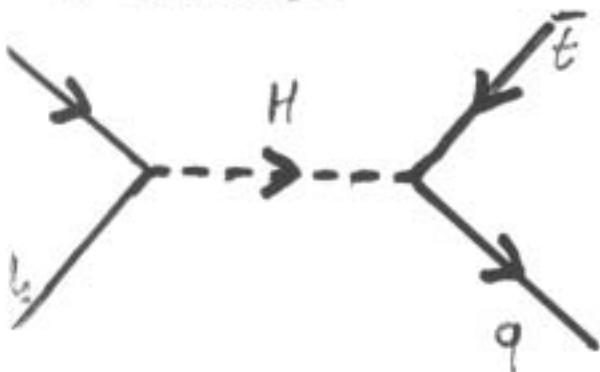
\Rightarrow the efficiency factor was underestimated of a factor

$$(3/2) [z_B(3K/2)/z_B(K)] \simeq 1.6$$

SCATTERINGS

scatterings involving t -quarks

s channel



The Kinetic Equations become:

$$\frac{dN_{N_1}}{dz} = -(D + S_t)(N_{N_1} - N_{N_1}^{eq})$$

$$\frac{dN_{B-L}}{dz} = -\epsilon_1 D(N_{N_1} - N_{N_1}^{eq}) - W_0 N_{B-L}$$

$$W_0 = \underbrace{W_{ID} + W_{DL=1}^{S_t}}_{\epsilon S_t + D} \propto \tilde{m}_1 = k m_1$$

Approximately:

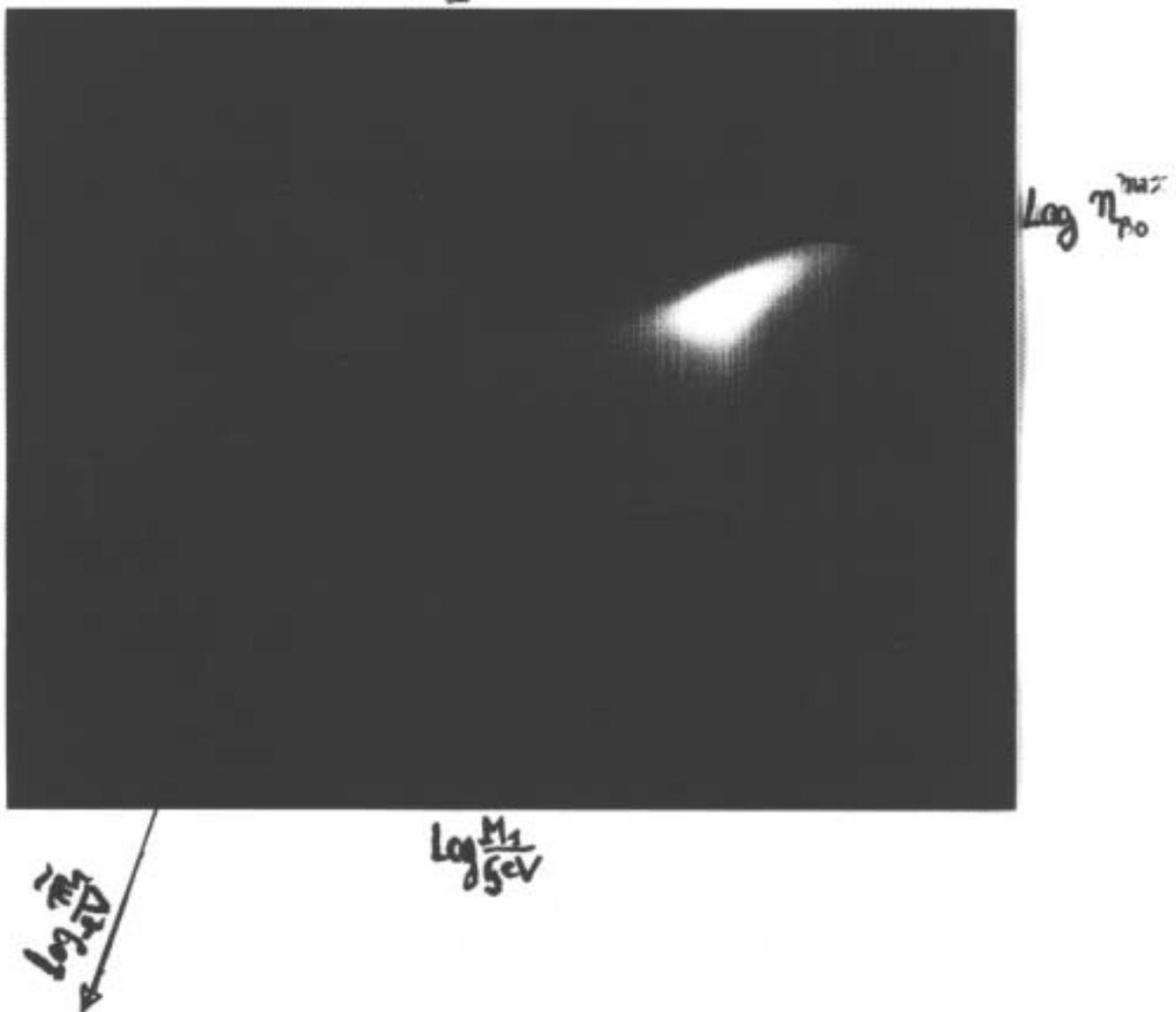
$$D \approx \frac{K z^2}{z + 15/8}$$

$$\rightarrow D + S_t \approx K z$$

A dependence on $\ln M_h/M_1$ can be neglected if $M_h/M_1 \ll 0.1$

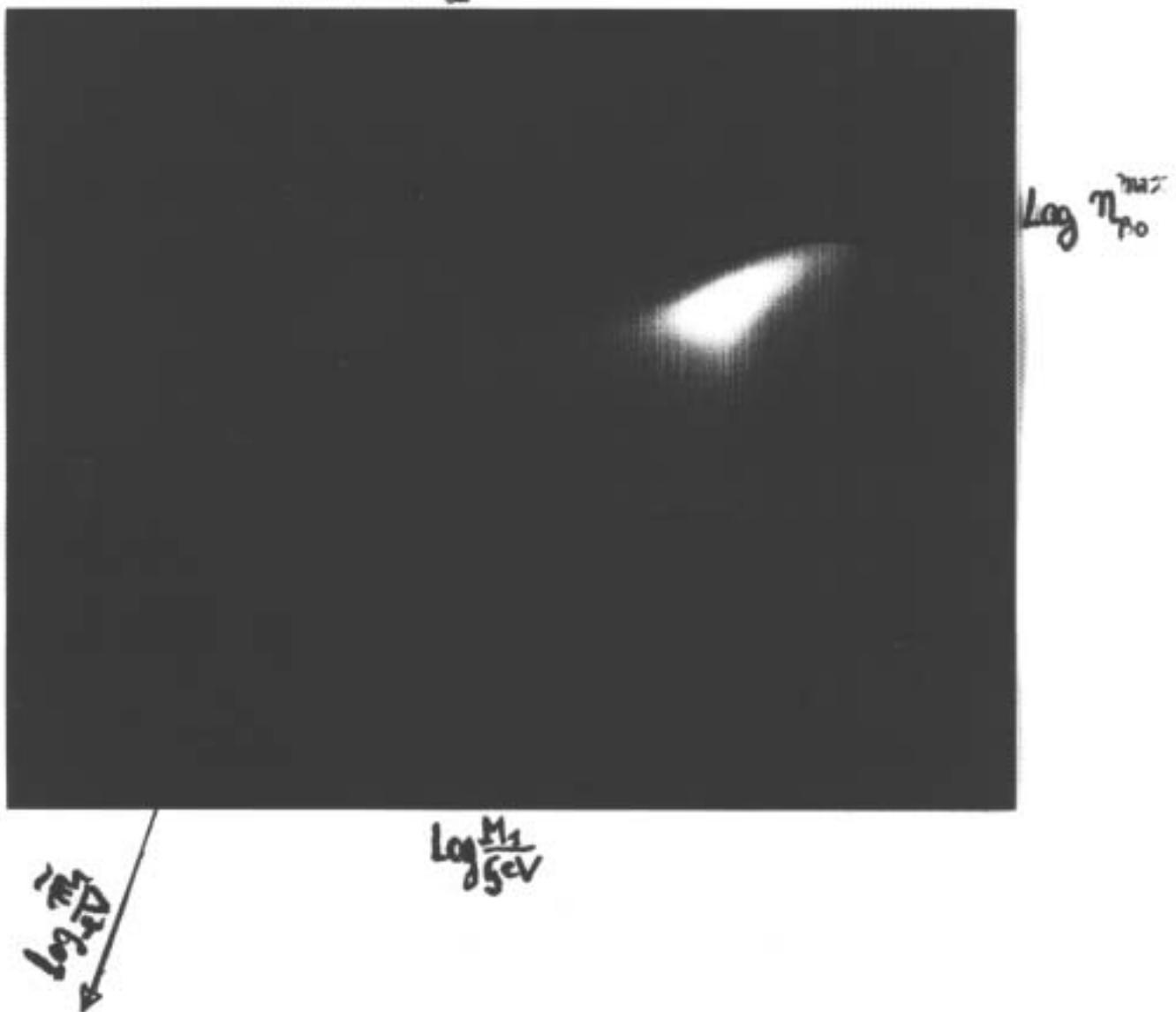
The leptogenesis mountain

(Buchmüller, PBS, Peimarcher)



The leptogenesis mountain

(Buchmüller, PBS, Peimarcher)



• • D+I.D.

$$\frac{M_1 = 10^{-5}}{M_2 = 10^{-10}} \left\{ \begin{array}{l} \text{D+ID+S} \\ \text{D+ID} \end{array} \right.$$



A small increase

$$N(k) \rightarrow$$

$$\approx 2N(k)$$

.... induces a increase in δr_f
[i.e. in the asym.]

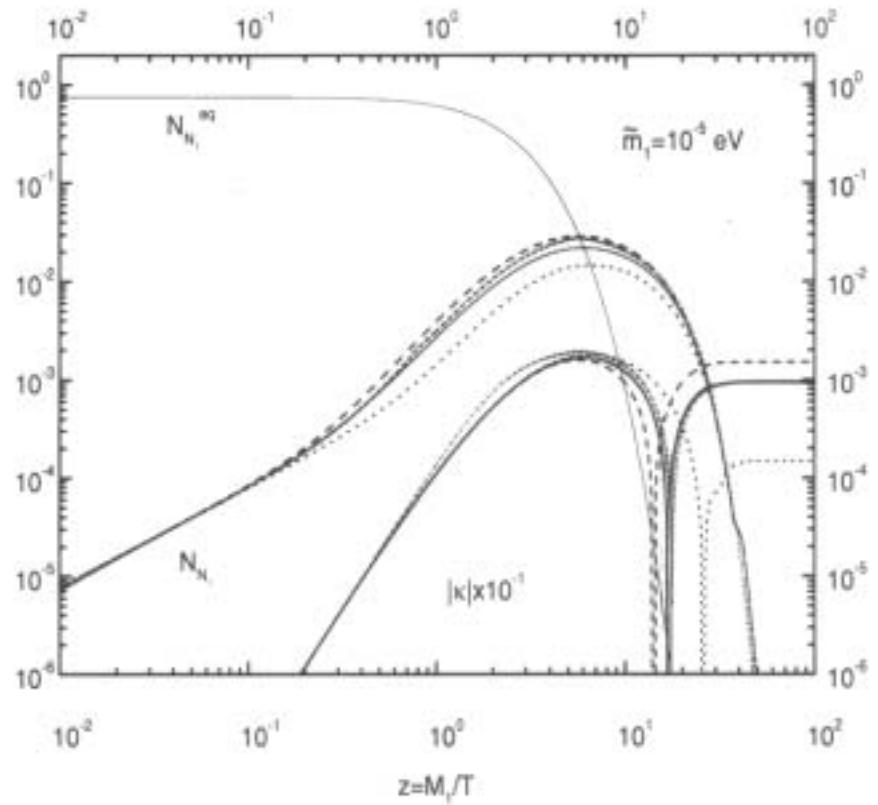
$$K \propto [r_f^+ - |K_f^-|]^{D+I.D.+S} \gg [r_f^+ - |K^-|]^{D+I.D.} \propto K^2$$



STRONG "INSTABILITY"

VERY CAREFUL DESCRIPTION IS NEEDED

Scatterings enhance the neutrino production



• BLACK : NUMERICAL RESULTS

$$--- M_h/M_1 = 10^{-10} \quad --- M_h/M_1 = 10^{-5} \quad \dots \quad N_h/N_1 = 1$$

• RED : ANALYTICAL RESULTS

$$---- N_{N_1}^{in} = N_{N_1}^{eq} = \frac{3}{4} \Rightarrow K_c = \frac{2}{K Z_B J(z_B)} \left[1 - e^{-\frac{K Z_B J(z_B)}{2}} \right]$$

$$\square \square \square \square \quad N_{N_1}^{in} = 0 \quad J(z_B) = 1 + \frac{2}{z_B} + \dots$$

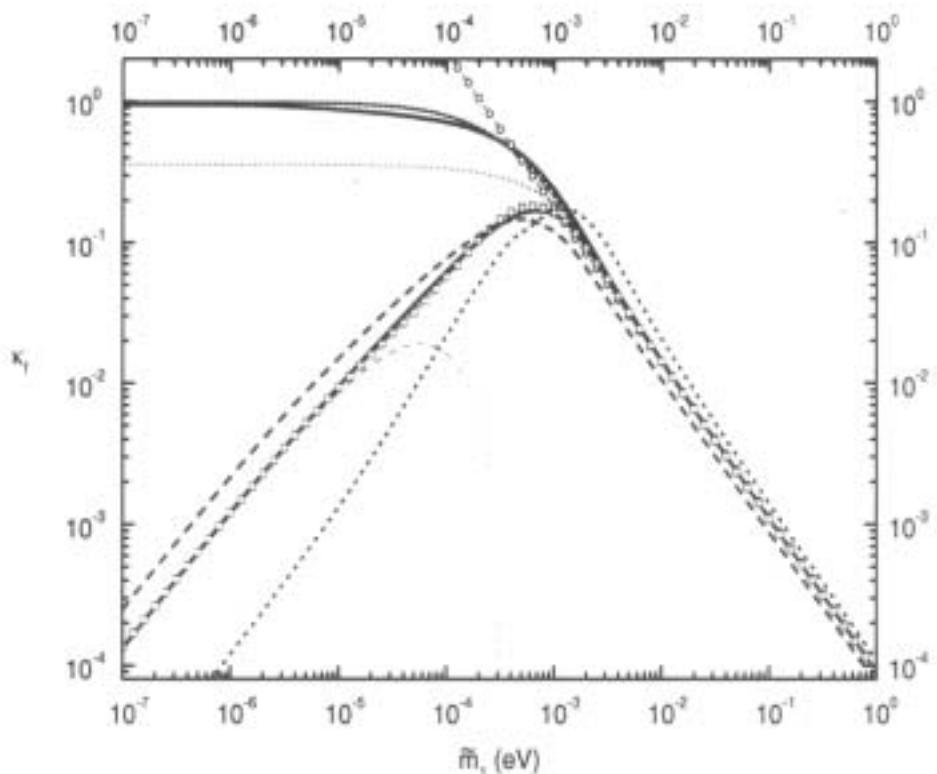
$$---- 0 < N_{N_1}^{in} < \frac{3}{4}$$

• BLUE $\circ \circ \circ$: POWER LAW
 $\beta = \frac{3}{2}$ 'OLD' $K_f \approx 0.9 \cdot 10^{-4} \left(\frac{eV}{m_1} \right)^{1.1}$

$$\overline{K_f} \approx 2.35 \left(\frac{eV}{m_1} \right)^{1.1}$$

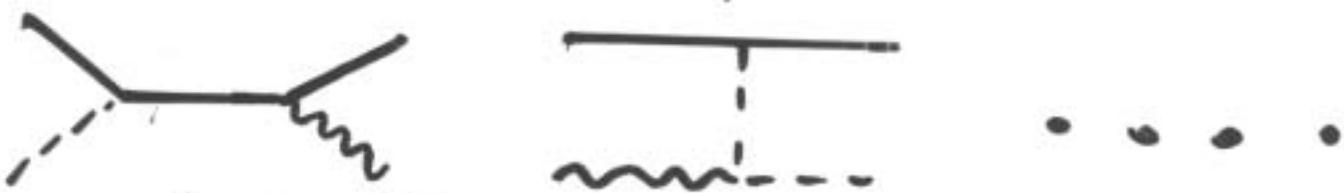
NEW ($\beta = 1$)

Final efficiency factor ($\beta = 3/2$) (regime of small $M_1 \sum_i m_{\nu_i}^2$)



RECENT DEVELOPMENTS AND THEORETICAL UNCERTAINTIES

- 1) $\beta = \frac{3}{2} \longrightarrow \beta = \frac{1}{2}$ ($\Delta L=2$ resonance removed)
- 2) - ACCOUNT OF THERMAL MASSES $\Rightarrow M_h \sim 0.4 \frac{M_1}{\Xi}$
 - SCALING (RGE) of top Yukawa coupling
 \Rightarrow strong suppression of scatterings ($\leq 2\text{ fm}$)
 The D+I.D. picture is almost recovered
- (Barbieri, Cremmelli, Strumia, Tettadris,
 Giudice, Notari, Strumia, Raidal, Riotto
 v3 March '03
 October '03)
- 3) - ACCOUNT OF SCATTERINGS INVOLVING GAUGE BOSONS
 (Pilaftsis, Underwood '0309 Giudice, Notari, Raidal, Riotto, Strumia 0310)



\Rightarrow "hybrid" result { weak wash-out regime: N_g production enhancement
 is back!
 strong wash-out regime: still suppression of $\Delta L=1$

- at high T Higgs decays contribute to the asymm. scattering wash-out
- CP asymmetry is dynamically evolving: $C_g = C_g [1 + f(T)]$

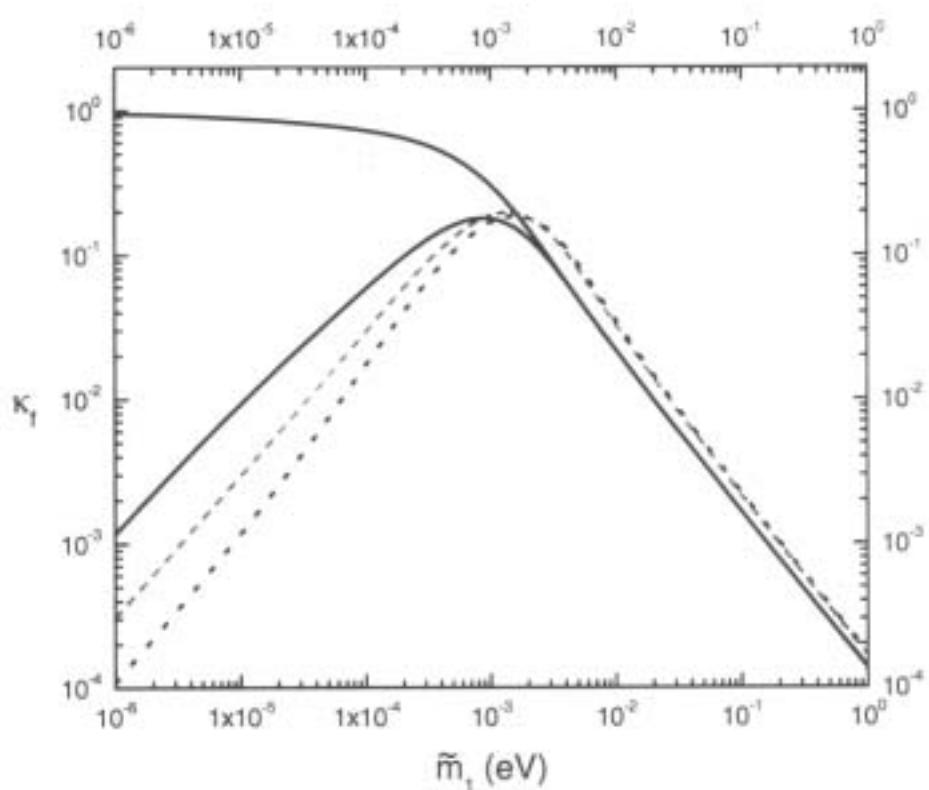
$$= \text{if } K \gg 1 \quad \left\{ k_f \approx 1.65 \left(\frac{M_1}{\Lambda_Y} \right)^{1.1} \right\} \quad \begin{array}{l} \text{(dynamically "power-law")} \\ \text{should be a poor fit} \end{array}$$

To be compared with: $\left\{ k_f = 9.35 \left(\frac{M_1}{\Lambda_Y} \right)^{1.2} \right\} \quad \text{if } K_f \approx (1/2) \left(\frac{M_1}{\Lambda_Y} \right)^{1.1}$

But... "spectator processes" (Buchmuller, Rummel) still not included

Final efficiency factor ($\beta = 1$)

(regime of small $M_1 \sum_i m_{\nu_i}^2$)



CMB constraint

(Buchmüller, PDD, Reimannacher '02)

$$V\text{MAP} + \text{SLOAN} \Rightarrow n_B^{\text{CMB}} = (6.3 \pm 0.3) \cdot 10^{-10}$$

$$\text{leptogenesis} \Rightarrow n_B \approx 10^{-2} |\epsilon_1| \cdot K_f$$

MAXIMUM
CP ASYMMETRY

$$\epsilon_1 = \epsilon_1^{\max} \cdot \sin \delta_L$$

MAXIMUM
BARYON ASYMMETRY

$$n_B^{\max} \approx 10^{-2} \epsilon_1^{\max} \cdot K_f$$

CMB
constraint:

$$\boxed{n_B^{\max} \geq n_B^{\text{CMB}}}$$

$$\epsilon_1^{\max} = 10^{-6} \frac{M_1}{10^{10} \text{GeV}} \cdot \beta(\tilde{m}_1, m_1)$$

$$0 \leq \beta \leq 1 \Rightarrow \text{it suppresses } \epsilon_1^{\max}$$

$$\beta(\tilde{m}_1, m_1) \approx \frac{m_{\text{dm}}}{m_1 + m_3} \cdot \left(1 - \frac{m_1}{\tilde{m}_1}\right) \quad (\tilde{m}_1 \geq m_1)$$

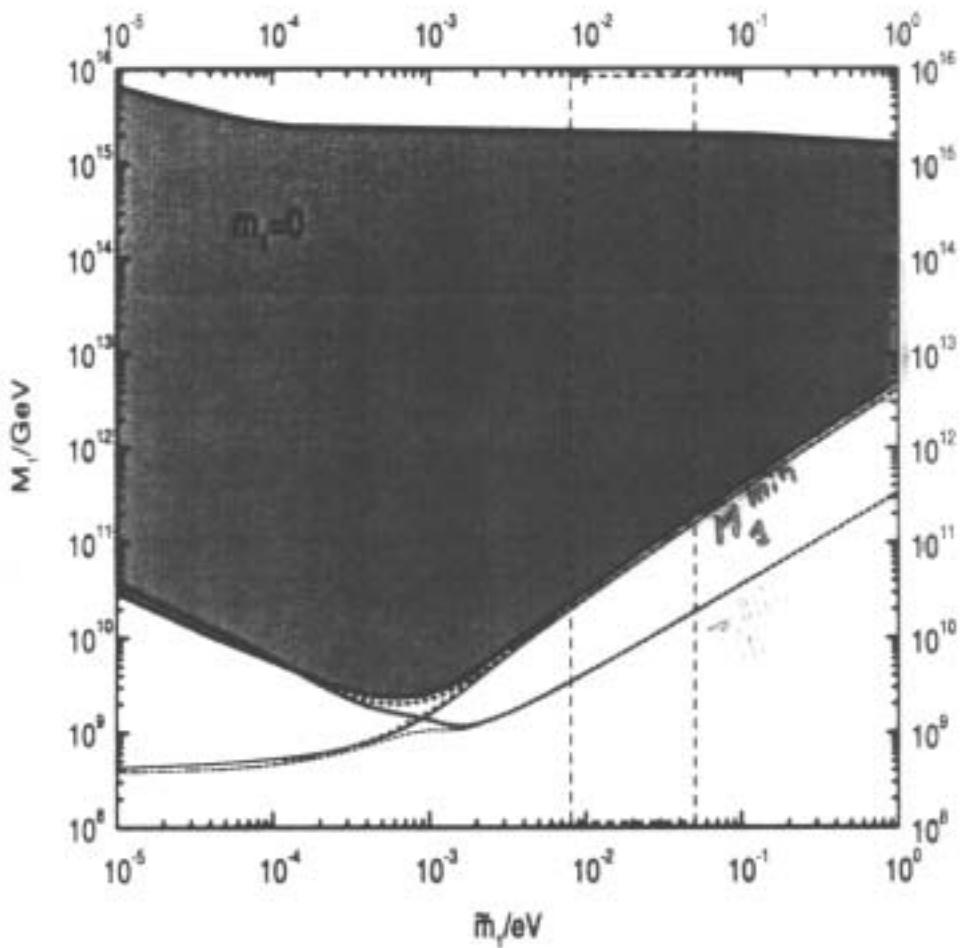
$$\begin{cases} \beta = 1 & \text{for } m_1 = 0 \\ \end{cases}$$

$$\begin{cases} K_f |_{m_1=0} > K_f |_{m_1 \neq 0} \end{cases}$$

For $m_1 = 0$ the allowed region ($n_B^{\max} \geq n_B^{\text{CMB}}$)
is the parameter

The allowed region

$$\eta_B^{\max} = \frac{3}{4} \frac{\alpha_{\text{ph}}}{f} \cdot \epsilon_1^{\max} \cdot k_f \geq n_B^{\text{CMB}}$$



$$M_1 = 0 \Rightarrow \epsilon_1^{\max} \approx 10^{-6} \left(\frac{M_1}{10^{10} \text{ GeV}} \right) \cdot \left(\frac{\sqrt{4m_{\text{dm}}^2}}{0.05 \text{ eV}} \right)$$

$$M_1 \ll 10^{14} \text{ GeV} \Rightarrow k_f = k_0(\tilde{m}_1) \Rightarrow \left\{ M_1 \geq M_1^{\min} = \frac{(M_1^{\max})^2}{k_0(\tilde{m}_1)} \right.$$

Lower bound on M_1

$$n_B^{\max} \simeq 10^{-2} \epsilon_1^{\max} \cdot K_f \geq n_B^{\text{CMB}}$$

$$\epsilon_1^{\max} \simeq 10^{-6} \cdot \frac{M_1}{10^{10} \text{GeV}} \cdot \left(\frac{m_{\text{ATH}}}{0.05 \text{eV}} \right)$$

$$\Rightarrow M_1 \geq \frac{6.4 \cdot 10^8 \text{GeV}}{K_f} \left(\frac{n_B^{\text{CMB}}}{c \cdot 10^{-10}} \right) \left(\frac{0.05 \text{eV}}{m_{\text{ATH}}} \right)$$

$$M_1 \geq \frac{(6.6 \pm 0.8) \cdot 10^8 \text{GeV}}{K_f} \geq \frac{4 \cdot 10^8 \text{GeV}}{K_f} \quad (30)$$

- initial thermal abundance ($N_{N_1}^{\text{in}} = N_{H_1}^{\text{eq}} = \frac{3}{4}$)

$$K_f^{\max} = 1 \text{ for } \tilde{m}_1 = 0 \Rightarrow M_1 \geq 4 \cdot 10^8 \text{GeV}$$

- initial vanishing abundance ($N_{N_1} = 0$)

$$K_f^{\max} \approx 0.77 \text{ for } \tilde{m}_1 \approx 10^{-3} \text{eV} \Rightarrow M_1 \geq 2 \cdot 10^8 \text{GeV}$$

- STRONG WASH-OUT REGIME

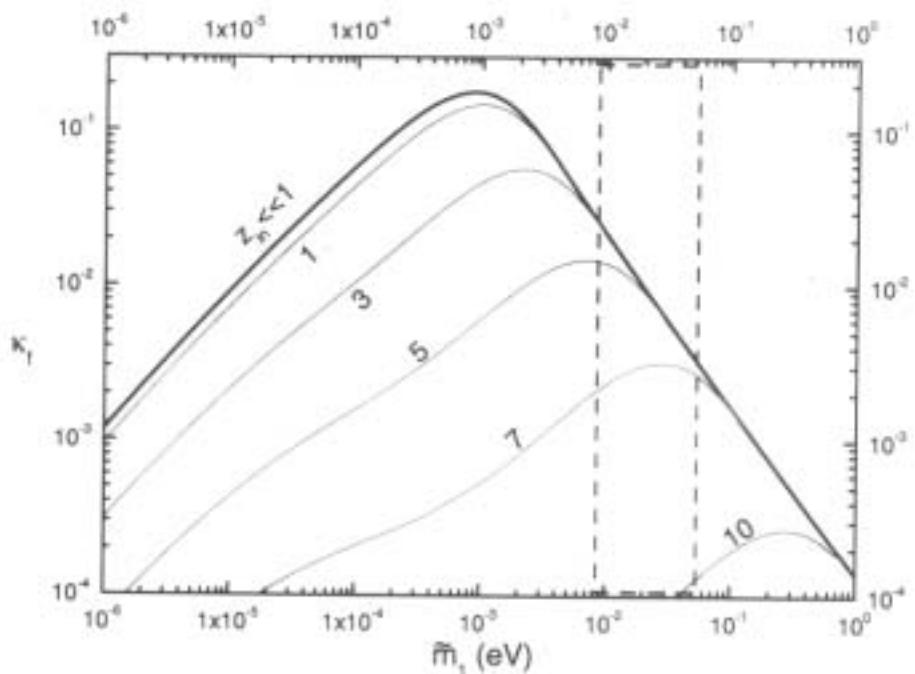
$$K_f \approx 1.35 \cdot 10^{-4} \left(\frac{1}{\tilde{m}_1} \right)^{1.1} \Rightarrow \left\{ \begin{array}{l} M_1 \geq 2 \cdot 10^{10} \text{GeV} \cdot \left(\frac{\tilde{m}_1}{0.01} \right) \\ \approx (10^{10} \div 10^{11}) \text{GeV} \\ (\tilde{m}_1 \in [m_{\text{SUSY}}, m_{\text{ATH}}]) \end{array} \right.$$

- DEGENERATE LEPTOGENESIS
(P. Crotts '97, P. Crotts, Underwood '03)

$$\epsilon_1^{\max} \rightarrow \epsilon_1^{\max} \cdot \xi \left(\frac{\Delta H_{21}}{M_1} \right) \Rightarrow M_1^{\min} \rightarrow \frac{M_1^{\min}}{\xi}$$

NOTE: M_1 LOWER BOUND IS SENSITIVE TO THEORETICAL UNKNOWN ξ

Lower Bound on T_{in}



Neutrino masses

$$m_3 \geq m_2 \geq m_1$$

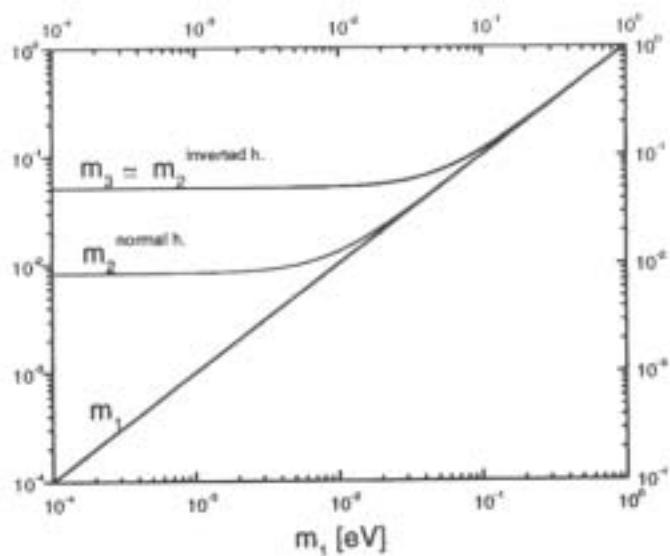
$$\begin{aligned}\Delta m_{\text{atm}}^2 &\simeq 2.5 \times 10^{-3} \text{ eV}^2 \\ \Delta m_{\text{LMA}}^2 &\simeq 7 \times 10^{-5} \text{ eV}^2\end{aligned}$$

Normal hierarchy

$$\begin{aligned}\Delta m_{32}^2 &= \Delta m_{\text{atm}}^2 \\ \Delta m_{21}^2 &= \Delta m_{\text{LMA}}^2\end{aligned}$$

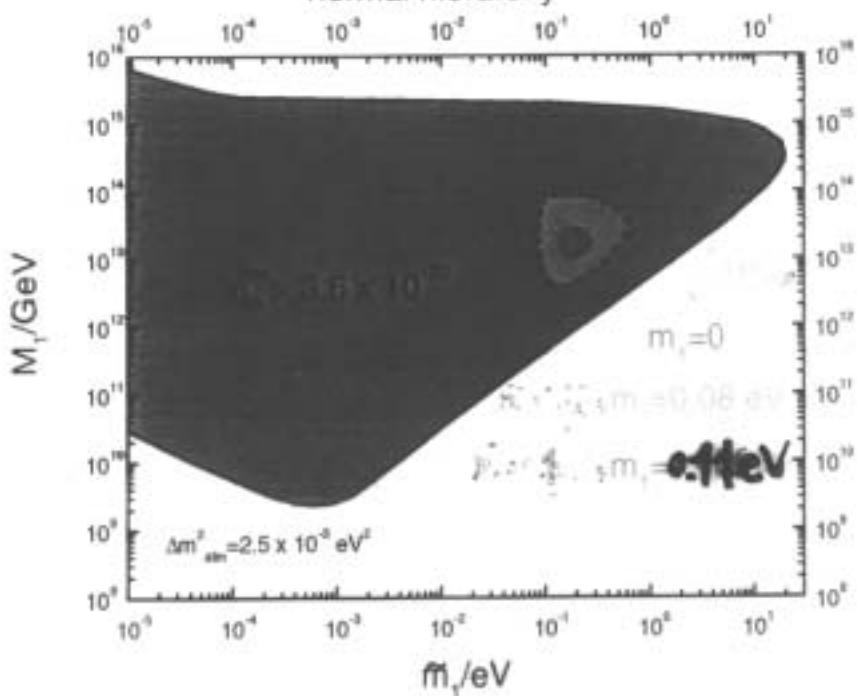
Inverted hierarchy

$$\begin{aligned}\Delta m_{32}^2 &= \Delta m_{\text{LMA}}^2 \\ \Delta m_{21}^2 &= \Delta m_{\text{atm}}^2\end{aligned}$$

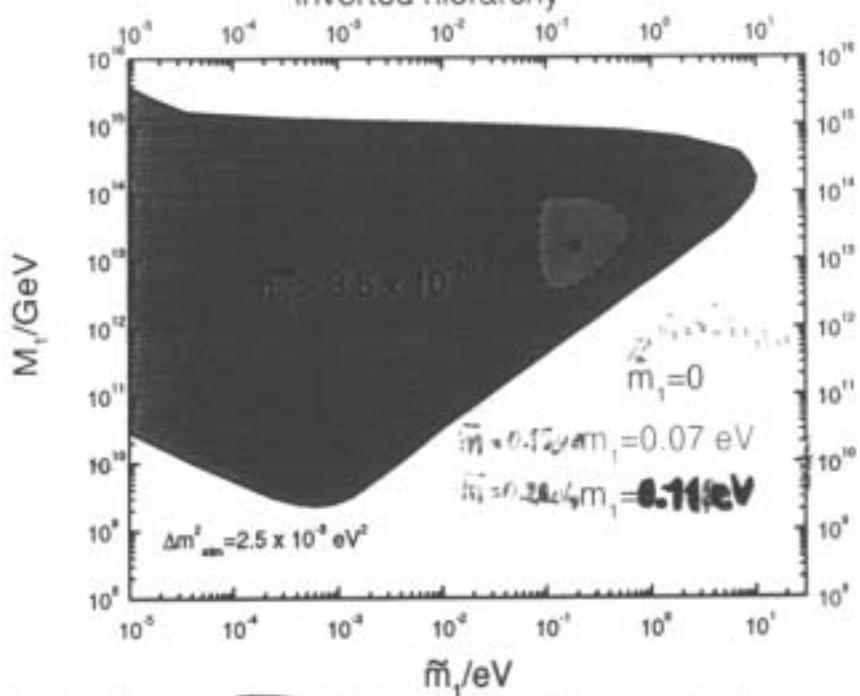


Numerical results

normal hierarchy



inverted hierarchy



$$\begin{aligned} \overline{m}_{\text{net}} &< 0.20 \text{ eV} \\ \overline{m}_{\text{light}} &< 0.21 \text{ eV} \end{aligned} \Rightarrow \boxed{|m_3| < 0.11 \text{ eV}}$$

Improves the previous one $|m_3| < 0.17 \text{ eV}$ (Buchmueller 2003, Riva 2006)

Boltzmann Equations

(Luty'92; Plumacher'97; Barbieri et al.'00; Buchmüller, PDB, Plümacher'02)

$$D, S, W_0, \Delta W \equiv \frac{\Gamma_D, \Gamma_S, \Gamma_{W_0}, \Gamma_{\Delta W}}{Hz}$$

$$z \equiv M_1/T, \quad D, S, W_0 \propto \bar{m}_1, \quad \Delta W \propto M_1 \bar{m}^2$$

$$\bar{m}^2 = m_1^2 + m_2^2 + m_3^2$$

$$= 3m_1^2 + \Delta m_{\text{atm}}^2 + 2\Delta m_{\text{sol}}^2 \quad (\text{n.h.})$$

$$= 3m_1^2 + 2\Delta m_{\text{atm}}^2 + \Delta m_{\text{sol}}^2 \quad (\text{i.h.})$$

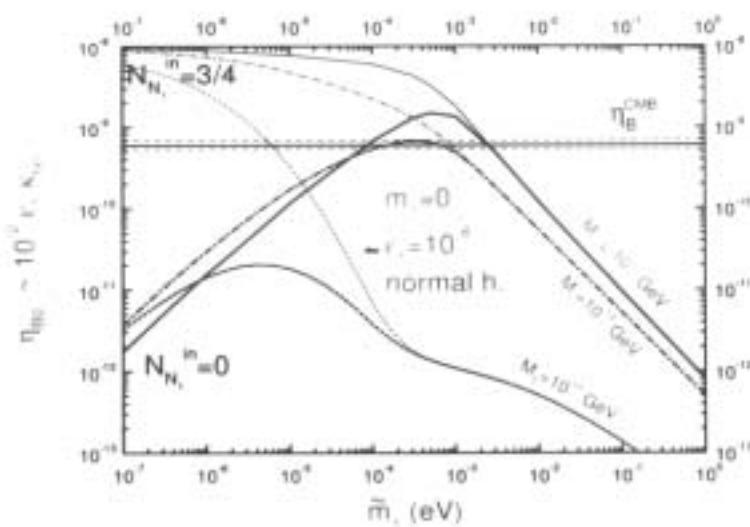
$$\begin{cases} \frac{dN_{N_1}}{dz} = -(D+S)(N_{N_1} - N_{N_1}^{\text{eq}}) \\ \frac{dN_{B-L}}{dz} = -\epsilon_1 D(N_{N_1} - N_{N_1}^{\text{eq}}) - (W_0 + \Delta W) N_{B-L} \end{cases}$$

$$N_e/N_{N_1}$$

$$T_{\eta_B} \gg T_A$$

$$N_{\nu_L} \ll N_{\nu_R}$$

$$M_1 \lesssim 10^{13} \text{ GeV} / (0.1 \text{ eV}/\bar{m})^2 \Rightarrow e^{- \int_1^\infty dz' \Delta W(z')} \ll 1$$



Analytical insight and stability of the bound

In the region where the bound applies:

$$\kappa_f(\tilde{m}_1, M_1 \bar{m}^2) = \kappa_f^0(\tilde{m}_1) e^{-\frac{12 W_\Delta}{z}} \frac{M_1}{10^{10} \text{GeV}} \left(\frac{\bar{m}}{\text{eV}} \right)^2$$

from this one can show that the 'coordinates' of the peak value of η_B^{\max} when $\eta_B^{\text{peak}} = \eta_B^{\text{CMB}}$ are:

$$\tilde{m}_1|_{\text{peak}} \simeq \bar{m}_{\max}$$

$$M_1|_{\text{peak}} \simeq 1.6 \times 10^{13} \text{GeV} \left(\frac{0.2 \text{eV}}{\bar{m}_{\max}} \right)^2$$

The value of \bar{m}_{\max} is slightly different for normal and inverted hierarchy:

$$\begin{aligned} (\bar{m}_{\max}^{\text{nor}})^2 &= (\bar{m}_{\max}^0)^2 - \frac{1}{8} m_{\text{atm}}^2 + \mathcal{O}(m_{\text{atm}}^4/\bar{m}_{\max}^4) \\ (\bar{m}_{\max}^{\text{inv}})^2 &= (\bar{m}_{\max}^0)^2 + \frac{7}{8} m_{\text{atm}}^2 + \mathcal{O}(m_{\text{atm}}^4/\bar{m}_{\max}^4) \end{aligned}$$

Dependence on the experimental quantities:

$$\begin{aligned} \bar{m}_{\max}^0 &\simeq 0.175 \text{ eV} \left(\frac{6 \times 10^{-10}}{\eta_B^{\text{CMB}}} \right)^{\frac{1}{4}} \left(\frac{m_{\text{atm}}}{m_0} \right)^{\frac{1}{2}} \\ \Rightarrow \delta \bar{m}_{\max}^0 &\simeq \frac{1}{4} \left(\delta \eta_B^{\text{CMB}} + \delta m_{\text{atm}}^2 \right) \end{aligned}$$

Using the MAP result ($\delta\eta_B^{CMB} \simeq 4\%$): $\Delta\bar{m}_{\max}^0 \simeq 0.01 \text{ eV}$
 $\Rightarrow \bar{m}_{\max}^0 < 0.205 \text{ eV} \Rightarrow m_1 < 0.115 \text{ eV} \quad (\sim 3\sigma)$
 theoretical uncertainty

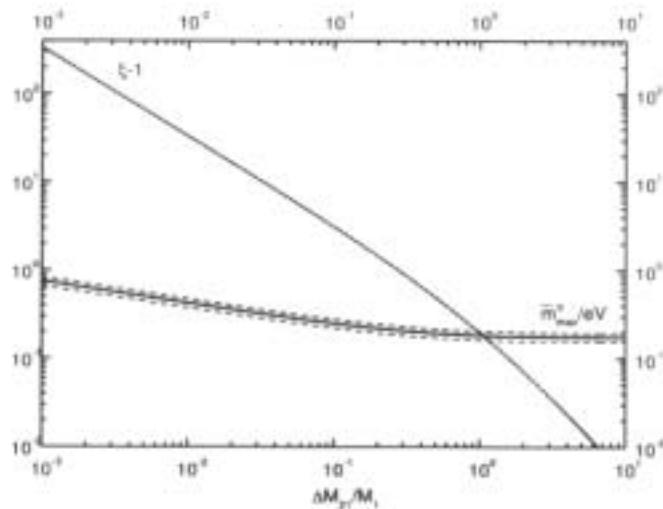
$$\bar{m}_{\max} \propto \left(\eta_B^{\text{exact}} / \eta_B^{\text{approx}} \right)^{1/4}$$

'degenerate' leptogenesis

if $\Delta M_{i1}/M_1 \lesssim 1$ ($i \neq 1$) then

$$\epsilon_1^{\max} \rightarrow \epsilon_1^{\max} \xi (\Delta M_{i1}/M_1), \quad \xi \geq 1$$

$$\Rightarrow \bar{m}_{\max}^0 < 0.205 \text{ eV} \xi^{1/4}, \quad m_1 \lesssim 0.11 \xi^{1/4} \text{ eV}$$



sleptogenesis: $m_1 < 0.105 \text{ eV}$

In conclusion.....: the bound is stable!

running of neutrino masses
 (Ratz et al) \Rightarrow the bound gets more stringent
 (at minimum 20%-30%)

A special case

$$M_3 = \frac{M^2}{m_1} ; M_2 = \frac{M^2}{m_2} ; M_1 = \frac{M^2}{m_3}$$

$\Rightarrow \xi \approx 7 \quad \xi^{1/4} \approx 1.6 \Rightarrow \boxed{m_1 \approx 0.16 \text{ eV}}$

Two opposite corrections

- $m_1 \lesssim 0.11 \text{ eV} \cdot \left(\frac{3/2}{\beta}\right)^{1/2}$ (Giudice et al '03)

$$\beta = \frac{3}{2} \rightarrow \beta = 1 \Rightarrow \boxed{m_1 \lesssim 0.12 \text{ eV}}$$

But...

- Running of neutrino masses
(Antusch, Kersten, Lindner, Ratz '03)

$$m_1 \lesssim 0.12 \text{ eV} \cdot \left(\frac{m_{ATM}}{0.05 \text{ eV}}\right)^{1/2}$$

The ATMOSPHERIC neutrino mass scale has to be evaluated at $\sim 10^{13} \text{ GeV}$

$$+20\% \Rightarrow m_1 \lesssim 0.145 \text{ eV} \quad \text{at } T \sim 10^{13} \text{ GeV}$$

\swarrow

$$\boxed{-40\% \quad \boxed{m_1 \lesssim 0.10 \text{ eV}}}$$

Upper bound "updated"

• $\beta = \frac{3}{2} \rightarrow \beta = 1$

\Rightarrow

Conclusions

- Leptogenesis is 'easy': analytical expressions not only provide a deep physical insight but they are also precise (the precision is less than the theoretical uncertainties)
- In the strong wash out regime leptogenesis works at its best and the analytical expressions too:
 - independence on the initial conditions
 - stability under variation of the physical assumptions
 - small theoretical uncertainties (conservatively factors $\mathcal{O}(1)$ on κ_f)
 - great precision of the analytical expressions
 - existence of a well defined $T_B \ll M_1 \Rightarrow$ non relativistic approximations work well (e.g. MB approximation)
- The fact that neutrino mixing data favour leptogenesis to lie in the strong wash-out regime, (but not too much: $K_{\text{lep}} = \mathcal{O}(10)$) makes leptogenesis very attractive and simple (it resembles closely Standard Big Bang Nucleosynthesis)
- potential old problem with the M_1 and T_{reh} lower bound, but the second one is alleviated of one order of magnitude;
- prediction: leptogenesis, in the minimal version we have considered, likes hierarchical neutrinos. In the case of quasi degenerate neutrinos playing with heavy RH degeneracies can help but requires a good amount of tuning.
A more natural way out would be possibly type two seesaw ... but at the expenses of predictivity.