ON THE CONNECTION BETWEEN LEPTOGENESIS & LOW-ENERGY CPV

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Outline

- LOW ENERGY "OBSERVABLE" CPV PHASES: δ, α_{21}, α_{31}.
- THE SEE-SAW MECHANISM
- LEPTOGENESIS AND LFV CHARGED LEPTON DECAYS
- ON THE CONNECTION BETWEEN CPV PHASES IN LEPTOGENESIS, LFV ℓ-DECAYS AND m_ν.
- TWO EXAMPLES:
  - A NORMAL HIERARCHICAL MODEL;
  - THE QUASI-DEGENERATE ν-MASS CASE.

CP-VIOLATION IN THE LEPTON SECTOR

For 3-ν mixing, U_{PMNS} can be parametrized as:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}} & 0 \\ 0 & 0 & e^{i(\alpha_{31}-\delta)} \end{pmatrix}$$

There are 3 CP-V PHASES: δ, α_{21}, α_{31}.

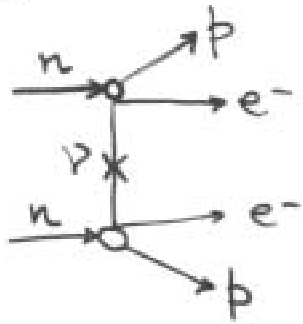
- the universal CP-V PHASE δ.

  It enters in ν-oscillations and it may be measured in future long-base line experiments.
  This requires: $\sin^2\theta_{13}$ not too small, to resolve degeneracies among different parameters ($\theta_{13}, \delta \leftrightarrow \theta'_{13}, \delta'$), to disentangle from matter effects.

- two Majorana CPV PHASES.

They are physical only if ν are Majorana particles. They do not enter in ν-oscillations but are in principle measurable in $\Delta L = 2$ processes and in particular in $(\beta\beta)_{0\nu}$-decay.
(e.g. Rodejohann; S.P., Petcov, Rodejohann; Barger et al.)

$(\beta\beta)_{0\nu}$-decay proceeds through the exchange of Majorana neutrinos in certain nuclei:



$\Delta L = 2$

$$\left[T_{0\nu}^{1/2}\right]^{-1} \propto |\langle m \rangle|^2 M^2_{\text{nuclear}}$$

$|\langle m \rangle|$ is the effective Majorana mass parameter:

$$|\langle m \rangle| = \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 e^{i\alpha_{21}} + m_3 |U_{e3}|^2 e^{i\alpha_{31}} \right|$$

$$\cong m_{\bar{\nu}_e} \left| \cos^2\theta_\odot + \sin^2\theta_\odot e^{i\alpha_{21}} \right|$$

for QD spectrum $(m_1 \simeq m_2 \simeq m_3 = m_{\bar{\nu}_e} \gg \Delta m^2_\odot, \Delta m^2_A)$

In principle, a measurement of $|<m>|$ combined with a better determination of the oscillation parameters and a precise measurement of m_1 would allow to establish if CP is violated and to constrain the CPViolating phases, α_{21}, α_{31}, once the spectrum is known. For ex., for the QD spectrum:

$$\sin^2 \alpha_{21}/2 \simeq \left(1 - \frac{|<m>|^2}{m_{\bar{\nu}e}^2}\right) \frac{1}{\sin^2 2\theta_\odot}.$$

The required accuracy in the determination of $|<m>|$ is affected by the uncertainty on the theoretical evaluation of the nuclear matrix elements.
Still there is a spread in $M_{nuclear}$ which amounts up to a factor of 3 in the values of $|<m>|$ derived from $T_{0\nu}^{1/2}$.
Hopefully (?) the nuclear matrix el. problems will be solved in the future: NSM, QRPA ...
A spread of 1.5-2 might allow to get information on the Majorana CPV phases.

Due to the experimental errors on the parameters and nuclear matrix elements uncertainties, determining that CP is violated in the lepton sector due to Majorana CPV phases is very difficult. (Barger et al.; S.P., Petcov, Rodejohann)

However it is possible if: (S.P., Petcov, Rodejohann)
i) an experimental error on $|{<}m{>}|$ $< 15\%$;
ii) a large value of $\tan^2\theta_\odot \gtrsim 0.55$;
iii) in the QD spectrum, a high value of neutrino masses: $m_{\bar{\nu}_e} \gtrsim 0.70$ eV or $\Sigma \gtrsim 1.5$ eV and an experimental error smaller than $(10 \div 15)\%$;
iv) $\alpha_{21,(32)} \sim (\pi/2 - 3\pi/4)$ or $\alpha_{21,(32)} \sim (5\pi/4 - 3\pi/2)$;
v) an uncertainty in the nuclear matrix elements which accounts to a factor ζ in $|{<}m{>}|$, $\zeta < 2$.

Suppose that we have measured the low energy CP-violating phases, δ, α_{21}, α_{31}, what can we infer on the physics beyond the SM, and, in particular, can we test the feasibility of

LEPTOGENESIS ?

THE SEE-SAW MECHANISM

We assume the existence of heavy right-handed Majorana neutrinos, ν_R, singlets under $SU(3) \times SU(2) \times U(1)$, with Majorana mass M_R.

$$\mathcal{L}_\nu = -Y_\nu \bar{\nu}_R L \cdot H - \frac{1}{2} \bar{\nu}_R^c M_R \nu_R + h.c.$$

The vev of H^0, v, provides a Dirac mass term: $m_D = Y_\nu v$.

$$\text{mass matrix:} \quad \mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

which is diagonalized: $\mathcal{D}_{M+m} = V \mathcal{M} V^T$.

Eigenstates:

- $\nu = U^T \nu_L + \nu_L^c U^*$ Majorana field!

$$d_m \cong - U^T \underbrace{Y_\nu^T M_R^{-1} Y_\nu}_{m_\nu} U v^2$$

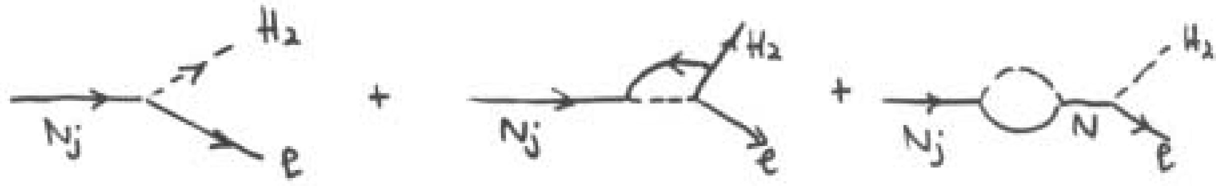
- $N \cong \nu_R + \nu_R^c$

$D_M \cong M_R$

LEPTOGENESIS

Fukugita, Yanagida
Covi, Roulet, Vissani
Buchmuller, Plumacher

The out-of-equilibrium decay of N_R can generate a lepton asymmetry which is then converted into y_B.
The decay asymmetry is given by the interference of tree-level and one-loop diagrams:



$$\varepsilon_i \equiv \frac{\Gamma(N_i \to \ell H) - \Gamma(N_i \to \ell^c H^c)}{\Gamma(N_i \to \ell H) + \Gamma(N_i \to \ell^c H^c)}$$

$$\simeq -\frac{1}{8\pi}\frac{1}{(Y_\nu Y_\nu^\dagger)_{ii}} \sum_{i \neq j} \mathrm{Im}(Y_\nu Y_\nu^\dagger)^2_{ij}\left[f\left(\frac{M_j^2}{M_i^2}\right) + g\left(\frac{M_j^2}{M_i^2}\right)\right]$$

$$f(x) = \sqrt{x}\left[1-(1+x)\ln\frac{1+x}{x}\right] \quad \text{and} \quad g(x) = \frac{\sqrt{x}}{1-x}$$

The lepton asymmetry is

$$y_L \equiv \frac{n_L - \bar{n}_L}{s} \propto \varepsilon_i$$

$$y_B = C\, y_L \quad \text{where } C \sim \mathcal{O}(1) \quad \left(\begin{array}{c} C = -0.35 \\ \text{in MSSM}\end{array}\right)$$

LFV ℓ-DECAYS: $\ell_i \to \ell_j \gamma$

Borzumati, Masiero
Hall, Kostelecky,
Hisano et al.
Casas, Ibarra
Tanimoto et al.

In the SM (+ ν masses and mixing) they are very much suppressed.

The supersymmetrization of the minimal see-saw model, contains new sources of L_i-violation:

$$W = W_0 - \frac{1}{2} \nu_R^{c\,T} M \nu_R^c + \nu_R^c Y_\nu L \cdot H_2$$

$$-\mathcal{L}_{soft} = (m_L^2)_{ij} \tilde{L}_i^\dagger \tilde{L}_j + (m_{eR}^2)_{ij} \tilde{e}_{Ri}^* \tilde{e}_{Rj} + \ldots + (A^e_{ij} H_d e_{Ri}^* \tilde{L}_j + \ldots + h.c.)$$

Even assuming universality at M_X scale:

$$(m_L^2)_{ij} = (m_{eR}^2)_{ij} = (m_{\tilde{\nu}})^2_{ij} = \delta_{ij} m_0^2$$

$$A^e = Y^e a_0 m_0$$

$$\vdots$$

the RG induce off-diagonal soft terms at low energy (in leading-log approx):

$$(m_L^2)_{ij} \cong -\frac{1}{8\pi} (3 m_0^2 + A_0^2) (Y_\nu^\dagger Y_\nu)_{ij} \log \frac{M_X}{M_R}$$

The resulting BR for $\ell_i \to \ell_j \gamma$ reads:

$$BR(\ell_i \to \ell_j \gamma) \sim \frac{\alpha^3}{G_F^2 m_S^8} \left| -\frac{1}{8\pi} (3 m_0^2 + A_0^2) \log \frac{M_X}{M_R} \right|^2 \tan^2\beta \left| (Y_\nu^\dagger Y_\nu)_{ij} \right|^2$$

PARAMETRIZATIONS OF Y_ν

- BIUNITARY PARAMETRIZATION: Davidson, Ibarra; Ellis et al.; S.P., Petcov, Rodejohann

$$Y_\nu = V_R^\dagger D_Y V_L$$

$$= \begin{pmatrix} 1 & & \\ & e^{i\alpha_R} & \\ & & e^{i\beta_R} \end{pmatrix} \tilde{V}_R^\dagger D_Y \begin{pmatrix} 1 & & \\ & e^{i\alpha_\omega} & \\ & & e^{i\beta_\omega} \end{pmatrix} \tilde{V}_L$$

$$2 \quad 1 \quad 2 \quad 1 \implies 6 \text{ PHASES}$$

- ORTHOGONAL PARAMETRIZATION: Casas, Ibarra; Ellis et al.; Branco et al.; S.P., Petcov, Yaguna

$$-U^* d_m U^\dagger \cong Y_\nu^T D_M Y_\nu v^2$$

$$-U^* d_m^{\frac{1}{2}} d_m^{\frac{1}{2}} U^\dagger \cong Y_\nu^T D_M^{\frac{1}{2}} \underbrace{R R^T}_{=1} D_M^{\frac{1}{2}} Y_\nu v^2$$

$$\implies Y_\nu \cong i \frac{D_M^{\frac{1}{2}}}{v} R \, d_m^{\frac{1}{2}} U^\dagger$$

R is an complex orthogonal matrix: $R = O e^{iA}$ (PPY)

with $A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$, $e^{iA} = 1 - \frac{\cosh r - 1}{r^2} A^2 + i A \frac{\sinh r}{r}$, $r = \sqrt{a^2+b^2+c^2}$

- TRIANGULAR PARAMETRIZATION: Branco et al.

$$Y_\nu = Y_\Delta U_Y, \qquad Y_\Delta = \begin{pmatrix} Y_{\Delta_1} & Y_{12} & Y_{13} \\ 0 & Y_{\Delta_2} & Y_{23} \\ 0 & 0 & Y_{\Delta_3} \end{pmatrix}$$

In the case of 3 ν_L and 3 ν_R:

- "high energy parameters"

D_M	3 real	
Y_ν	9 real	$9-3=6$ phases
	12 real	6 phases

- "parameters accessible at low energy"

D_m	3 real : $\Delta m^2_\odot$, Δm^2_A, m_1
U_{PMNS}	3 angles: $\theta_\odot$, θ_A, θ_{13}
	3 phases: δ, α_{21}, α_{31}
	9 parameters

9 parameters are missing of which 3 phases.

① LEPTOGENESIS

$$Y_\nu Y_\nu^\dagger = \phi^* \tilde{V}_R^\dagger D_Y^2 \tilde{V}_R \phi$$
$$\cong D_M^{1/2} R \frac{D_m}{v^2} R^\dagger D_M^{1/2}$$

i $R = R(\varphi_R)$

ii U_{PMNS} does not enter explicitly in leptogenesis !

② LFV ℓ-DECAYS

$$Y_\nu^\dagger Y_\nu = \tilde{V}_L D_Y^2 V_L$$
$$\cong U D_m^{1/2} R^\dagger D_M R D_m^{1/2} U^\dagger v^2$$

③ m_ν

$$m_\nu \cong - V_L^T D_Y V_R^* D_M^{-1} V_R^\dagger D_Y V_L$$
$$= U^* D_m U^\dagger$$

② and ③ $\Longrightarrow$ $U_{PMNS}(\underset{\uparrow \text{ leptogenesis}}{\varphi_R}, \varphi_L, \varphi_W)$!

There is no direct connection between leptogenesis and low energy CPV BUT many models allow for it.

THE CASE OF A HIERARCHICAL SPECTRUM

S.P., Petcov, Rodejohann
PRD

Assumptions: $M_1 \ll M_2 \lll M_3$

$dy_1 \ll dy_2 \ll dy_3$

$s_{1L,R} \sim 10^{-1} \gg s_{2L,R} \sim 10^{-2} \gg s_{3L,R}$.

$$Y_\nu = V_R^{+} D_Y V_L \cong \begin{pmatrix} -dy_2' s_{1L} s_{1R} & -dy_2' s_{1R} & dy_3' (s_{1R} s_{2R} - e^{i\delta_R} s_{3R}) \\ e^{-i\alpha_R} dy_2' s_{1L} & e^{-i\alpha_R} dy_2' & e^{-i\alpha_R} dy_3' s_{2R} \\ e^{-i(\beta_R+\delta_R-\delta_L)} dy_3' s_{3L} & e^{-i(\beta_R+\delta_R)} dy_3' s_{2L} & e^{-i(\beta_R+\delta_R)} dy_3' \end{pmatrix}$$

$dy_2' = dy_2 e^{i\alpha_W}$ $\quad dy_3' = dy_3 e^{i\beta_W}$

— o —

LFV- ℓ DECAYS.

$\mu \to e\gamma$: $(Y_\nu^+ Y_\nu)_{12} \simeq y_2^2 s_{1L}$

$\tau \to e\gamma$: $(Y_\nu^+ Y_\nu)_{13} \simeq y_3^2 s_{3L} e^{i\delta_L}$

$\tau \to \mu\gamma$: $(Y_\nu^+ Y_\nu)_{32} = y_3^2 s_{2L}$

$\Rightarrow$ Typically, $BR(\tau \to \mu\gamma) \gg BR(\tau \to e\gamma) \gg BR(\mu \to e\gamma)$

LEPTOGENESIS

$$\mathrm{Im}\,(Y_\nu Y_\nu^\dagger)^2_{12} \cong (y_2^2 + y_3^2\, s_{2R}^2)^2\, s_{1R}^2 \quad \sin 2\alpha_R$$

$$\mathrm{Im}\,(Y_\nu Y_\nu^\dagger)^2_{13} \cong y_3^2\, s_{1R}^2 s_{2R}^2 \quad \sin 2(\beta_R + \delta_R)$$

$$\varepsilon_i \simeq -\frac{3}{16\pi}\left((y_2^2 + s_{2R}^2 y_3^2)\sin 2\alpha_R \frac{M_1}{M_2} + \frac{y_3^4 s_{2R}^2}{y_2^2 + y_3^2 s_{2R}^2}\sin 2(\beta_R+\delta_R)\frac{M_1}{M_3}\right)$$

Having $Y_B \sim 6\times 10^{-10}$ requires $y_3 v = 10^2$ GeV; $M_2 \sim 10 M_1 \sim 10^{10}$ GeV

— o —

LOW - ENERGY CPV : δ, α_{21}, α_{31}

$$(\beta\beta)_{0\nu} : \quad \langle m\rangle \simeq \left(y_2^2 s_{1L}^2\, e^{2i\alpha_W}\left(\frac{s_{1R}^2}{M_1} + \frac{e^{2i\alpha_R}}{M_2}\right)\right) + \dots$$

$|\langle m\rangle|$ depends on α_R, the leptogenesis phase.

$$\delta: \qquad J_{CP} \propto 10^{-2} \sin(\alpha_W - (\beta_W + \delta_L) + 2\alpha_R - 2(\beta_R + \delta_R))$$

No direct connection between the leptogenesis phases and δ can be established.

QUASI-DEGENERATE ν MASS-SPECTRUM

S.P., Petcov, Yaguna
PLB

Assumptions: $m_1 \simeq m_2 \simeq m_3 \gg \Delta m^2_\odot, \Delta m^2_A$

$D_M \cong M_R \cdot \mathbb{1}$ + small deviations

We use the orthogonal parametrization:

$$Y_\nu \simeq \frac{i}{v} D_M^{\frac{1}{2}} R D_m^{\frac{1}{2}} U^+ = \frac{i}{v} D_M^{\frac{1}{2}} e^{iA} D_m^{\frac{1}{2}} U^+$$

LEPTOGENESIS

$$Y_\nu Y_\nu^+ = \frac{D_M D_m}{v^2} e^{2iA}$$

$$\varepsilon_1 = \varepsilon_2 \simeq \frac{1}{\pi} \frac{D_m D_M}{v^2} \frac{abc}{M_1 - M_2}$$

$$\varepsilon_3 \simeq -\frac{2}{\pi} \frac{D_M D_m}{v^2} \frac{abc}{M_1 - M_3}$$

$$\frac{n_B}{s} \simeq 1.4 \times 10^{-8} \left(\frac{m_\nu}{0.1\,\text{eV}}\right)\left(-\frac{abc}{M_1 - M_2}\right)\left(B_r^{(1)} + B_r^{(2)}\right)$$

$$\Longrightarrow \quad abc \simeq 10^{-5}$$

LFV ℓ-DECAYS:

$$Y_\nu^\dagger Y_\nu \sim \frac{D_M D_m}{v^2} \left(U e^{2iA} U^\dagger \right)$$

For real R (A=0), the BR are strongly suppressed (Casas & Ibarra, Tanimoto et al.) for QD neutrinos.

$$\left(Y_\nu^\dagger Y_\nu\right)_R = \frac{D_M}{v^2}\left(U_{e2} U_{e'2}^* \frac{\Delta m_\odot^2}{2 D_m} + U_{e3} U_{e'3}^* \frac{\Delta m_A^2}{2 D_m}\right)$$

For R complex, there is an enhancement:

$$\left(Y_\nu^\dagger Y_\nu\right)_{12} \propto \frac{D_M d_m}{v^2} \Big[-a \left(c_{12}^2 e^{i\alpha_{21}} + s_{12}^2 e^{-i\alpha_{21}}\right) c_{23} - e^{i\alpha_{31}} s_{23} \left(b\, c_{12} + c\, s_{12} e^{-i\alpha_{21}}\right) \Big]$$

Analogously for $\left(Y_\nu^\dagger Y_\nu\right)_{13}$ and $\left(Y_\nu^\dagger Y_\nu\right)_{23}$.

R real $\left|\left(Y_\nu^\dagger Y_\nu\right)\right|_{12}^2 \sim \frac{M_R^2 m_\nu^2}{v^4} \begin{cases} 6 \times 10^{-6} & s_{13} = 0.2 \\ 1.4 \times 10^{-8} & s_{13} = 0 \end{cases}$

R complex $\left|\left(Y_\nu^\dagger Y_\nu\right)\right|_{12}^2 \sim \frac{M_R^2 m_\nu^2}{v^4} \begin{cases} 0.34 & (a,b,c) = (0.2, -0.4, 0.5) \\ 0.81 & (a,b,c) = (0.4, 0.3, 0.2) \end{cases}$

CONCLUSIONS

- CPV IN THE LEPTON SECTOR IS PARAMETRIZED BY THE δ PHASE AND 2 MAJORANA PHASES. THEY ARE MEASURABLE, IN PRINCIPLE, IN ν-OSCILLATIONS (δ) AND $\Delta L = 2$ PROCESSES (α_{21}, α_{31}).

- IN THE CONTEXT OF THE MINIMAL SEE-SAW MECHANISM, LEPTOGENESIS & LFV ℓ-DECAYS MAY PROVIDE ADDITIONAL INFORMATION ON THE SEE-SAW PARAMETERS.

- - U_{PMNS} DOES NOT ENTER ESPLICITLY IN LEPTOGENESIS.
  - $U(\varphi_R, \varphi_L, \varphi_W)$ ($\varphi_R \equiv \alpha_R, \beta_R, \delta_R$, the leptogenesis phases)
  - THERE IS NO DIRECT CONNECTION BETWEEN φ_R AND $\delta, \alpha_{21}, \alpha_{31}$ BUT MANY MODELS ALLOW FOR IT.