A Possible Unconventional Origin of Neutrino Mass Differences

Quantum Gravity Decoherence, CPT Violation, Dark Energy and Neutrino Mass differences ?

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Fifty Years from the neutrino discovery

OUTLINE

- QUANTUM GRAVITY (QG) AS A DECOHERENING "MEDIUM" AND CPT VIOLATION .
- DARK ENERGY and Neutrino Mass difference: FUN WITH QUANTUM FIELD THEORIES WITH MIXING
- NEUTRINOS in (stochastically fluctuating) media (MSW effect revisited, "fake" CPT Violation and "decoherence" due to matter effects)
- QUANTUM-GRAVITY-INDUCED MSW EFFECT**?** A Viable (?) scenario
- GLOBAL NEUTRINO DATA FITS, **including LSND and KAMLAND** and possible QG scenaria: Work in progress.

CPT THEOREM

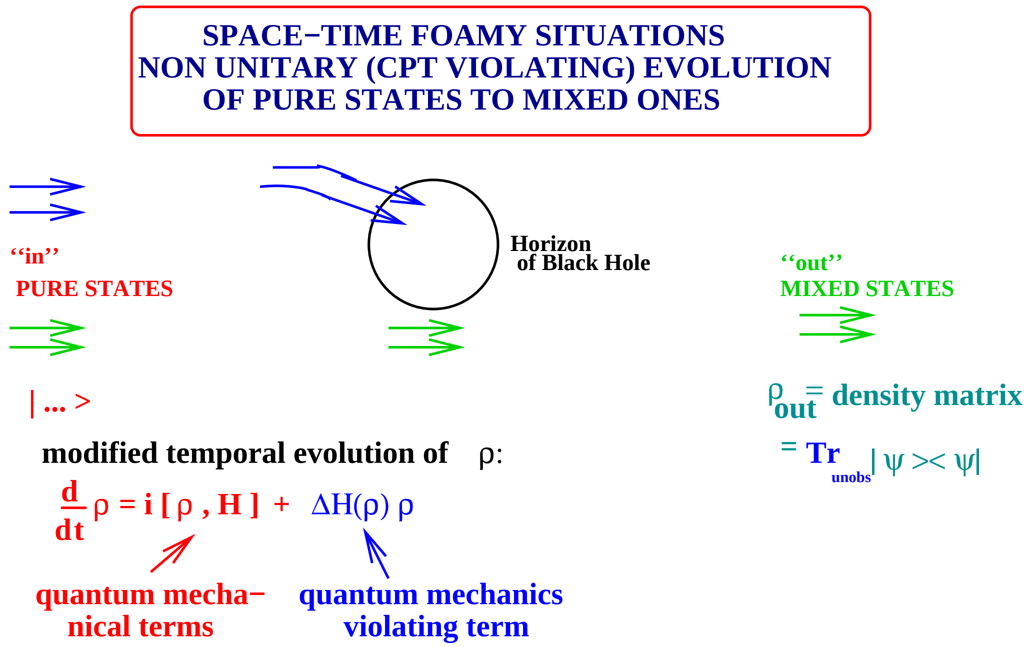
C(harge) -P(arity=reflection) -T(ime reversal) INVARIANCE is a property of any quantum field theory in Flat space times which respects:
(i) Locality, (ii) Unitarity and (iii) Lorentz Symmetry.

Theories with HIGHLY CURVED SPACE TIMES , of space time boundaries of black-hole horizon type, may violate (ii) &/or (iii), sometimes (i) and hence CPT.

e.g. SPONTANEOUS BREAKING OF LORENTZ SYMMETRY, OR
SPACE-TIME FOAMY SITUATIONS IN SOME QUANTUM GRAVITY MODELS INDUCING DECOHERENCE.

FOAM AND UNITARITY VIOLATION

SPACE-TIME FOAM: Quantum Gravity SINGULAR Fluctuations (microscopic (Planck size) black holes etc) **MAY** imply: pure states $\longrightarrow$ mixed



$$\rho_{out} = \mathrm{Tr}_{\mathrm{unobs}}|out >< out| = \$ \ \rho_{in},$$

$\$ \neq SS^{\dagger}$, $S = e^{iHt}$ =scattering matrix,

$\$$=**non invertible**, unitarity lost in effective theory.

BUT...HOLOGRAPHY can change the picture

(Strings in anti-de-Sitter space times (Maldacena, Witten), Hawking 2003- superposition of space-time topologies (Quantum Gravity) (but in Euclidean space time) may solve info-problem?: not quite sure (in QG) if the BH is there)

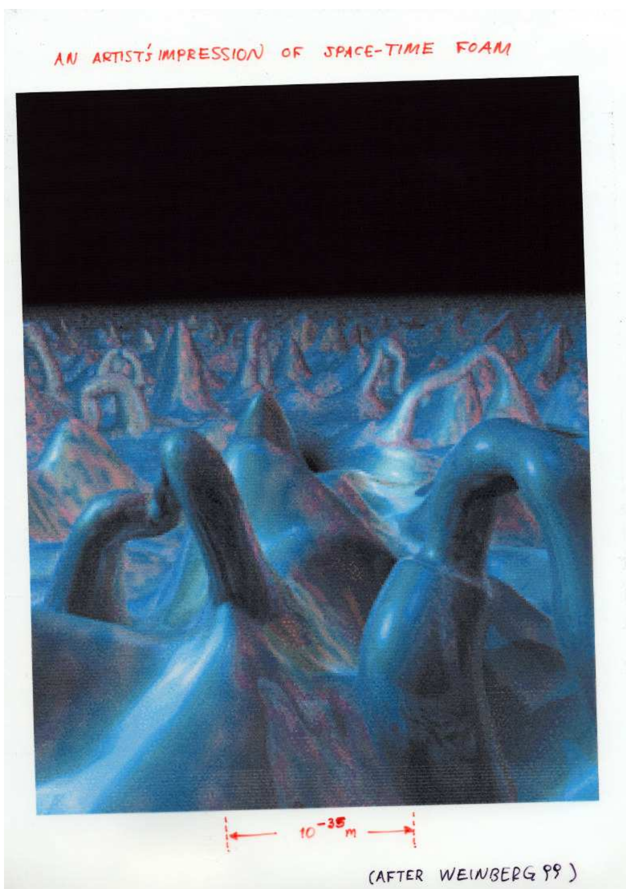
BUT NO PROOF AS YET ... OPEN ISSUE

SPACE-TIME FOAM and Intrinsic CPT Violation

A THEOREM BY R. WALD (1980):

If $ $\neq S\ S^\dagger$, then CPT is violated, at least in its strong form, i.e. a quantum mechanical CPT operator Θ , acting on ρ, is ill-defined.

NB: DISTINCT case from CPT Violation in Hamiltonian, i.e. $[\Theta, H] \neq 0$.



COSMOLOGICAL CPTV?

(NM, hep-ph/0309221)

Recent Astrophysical Evidence for Dark Energy (acceleration of the Universe (SnIA), CMB anisotropies (WMAP...))

Best fit models of the Universe consistent with non-zero **cosmological constant** $\Lambda \neq 0$ (de Sitter)

Λ-universe will eternally accelerate, as it will enter in an inflationary phase again: $a(t) \sim e^{\sqrt{\Lambda/3}t}$, $t \to \infty$, there is **cosmological Horizon**.

Horizon implies incompatibility with S-matrix & decoherence: no proper definition of asymptotic state vectors, environment of d.o.f. crossing the horizon (c.f. dual picture of black hole, now observer is inside the horizon).

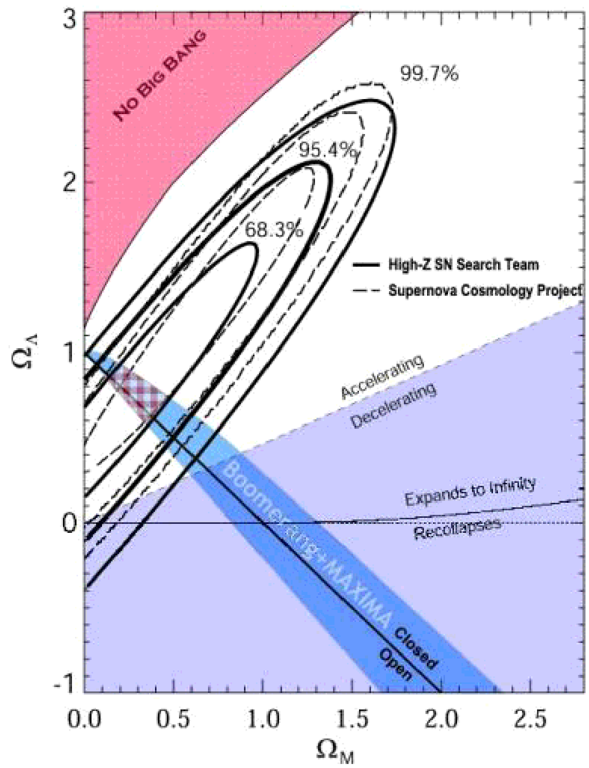
Theorem by Wald on \$-matrix and CPTV: CPT is violated due to $\Lambda > 0$ induced decoherence:

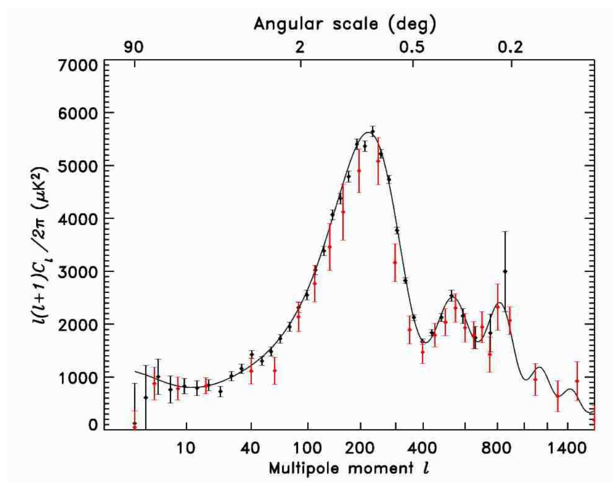
$$\partial_t \rho = i[\rho, H] + \frac{\Lambda}{M_P^3}[g_{\mu\nu}, [g^{\mu\nu}, \rho]]$$

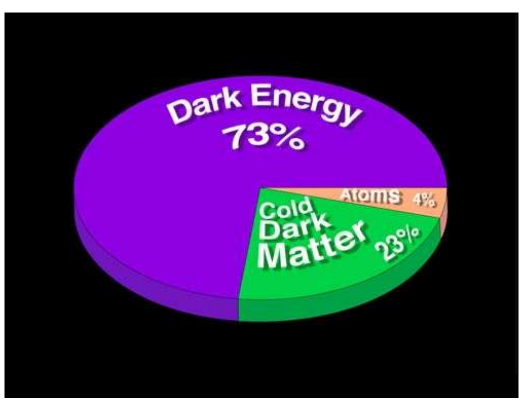
Tiny cosmological CPTV effects, but detected through Universe acceleration!

Evidence for Dark Energy

WMAP improved results on CMB: $\Omega_{\text{total}} = 1.02 \pm 0.02$, high precision measurement of secondary (two more) acoustic peaks (c.f. new determination of Ω_b). Agreement with SnIa Data. Best Fit : $\Omega_\Lambda = 0.73$, $\Omega_{\text{Matter}} = 0.27$







Dark Energy and ν mass differences

DOES Λ HAVE ANYTHING TO DO WITH ν?

NUMEROLOGY: Observed value of Λ today is $\Lambda \sim (\Delta m^2)^2$, $\Delta m^2 \sim 10^{-5}\mathrm{eV}^2$ **observed neutrino mass differences.**

Deeper connection or just coincidence ?

In this talk I will present an idea according to which:

- **If** Fock space Quantization is applied to **some** field-theoretic modes in theories with mixing (Blasone and Vitiello), then (Blasone et al. 2004, Barenboim, NM 2004)

$$\Lambda \propto \sin^2\theta(\Delta m^2)^2$$

- The origin of Δm^2 could be due to quantum gravity space-time foam (decoherence) (Barenboim, NM).

NB: The two items above could NOT be valid independently of one another though.

First item implies strong CPTV (Wald) $\rightarrow$ incompatible with local Lorentz invariant unitary quantum field theories. But if Δm^2 is precisely due to **decohered low-energy ν modes**, interacting with a **space time foam**, then compatibility **may** be restored. To be discussed next...

Dark Energy and Δm^2

Flavour space and Quantization: some problems

Quantum field theory (QFT) requires infinite volume limit. In contrast to quantum mechanical treatment of fixed volume (Pontecorvo), the neutrino *flavour* states are *orthogonal* to the *energy* eigenstates.

They define two inequivalent vacua related to each other by a *non unitary* transformation $G^{-1}(\theta, t)$ (Blasone, Vitiello 1995):

$$|0(t)\rangle_f = G_\theta^{-1}(t)|0(t)\rangle_m,$$

where θ is the mixing angle, t is the time, and the suffix f(m) denotes flavour(energy) eigenstates.
$G_\theta(t) = \exp\left(\theta \int d^3x [\nu_1^\dagger(x)\nu_2(x) - \nu_2^\dagger(x)\nu_1(x)]\right)$.

Bogolubov transformation connecting the creation and annihilation operator coefficients appearing energy or flavour eigenstates. Of the two Bogolubov coefficients concentrate on the one expressing condensate content of the flavour vacuum, $V_{\vec{k}} = |V_{\vec{k}}|e^{i(\omega_{k,1}+\omega_{k,2})t}$, $\omega_{k,i} = \sqrt{k^2 + m_i^2}$. with $_f\langle 0|\alpha^{r\dagger}_{\vec{k},i}\alpha^r_{\vec{k},i}|0\rangle_f =_f \langle 0|\beta^{r\dagger}_{\vec{k},i}\beta^r_{\vec{k},i}|0\rangle_f = \sin^2\theta|V_{\vec{k}}|^2$ in the two-generation scenario. For three generations there are various V_{ij}.

Properties of Flavour Condensate

$|V_{\vec{k}}| = 0$ for $m_1 = m_2$, has a maximum at $k^2 = m_1 m_2$, and for $k \gg \sqrt{m_1 m_2}$

$$|V_{\vec{k}}| \sim \frac{(m_1 - m_2)^2}{4|\vec{k}|^2}, \quad k \equiv |\vec{k}| \gg \sqrt{m_1 m_2}$$

Flavour vacuum $|0\rangle$, is the correct one to be used for vacuum energy contributions, since otherwise the probability is not conserved (Blasone, Henning, Vitiello 1999).

There are modifications in the oscillation probability, experimentally testable in principle... (Blasone, Henning, Vitiello)

Cosmological Constant and Δm^2

The energy-momentum tensor $T_{\mu\nu}$ of a Dirac fermion field in the Robertson-Walker space-time background can be calculated straightforwardly. The flavour-vacuum average of T_{00} is:

$$
\begin{aligned}
&{}_f\langle 0|T_{00}|0\rangle_f = \langle \rho_{\rm vac}^{\nu-{\rm mix}}\rangle \eta_{00} \equiv \Lambda \eta_{00} \\
&= \sum_{i,r}\int d^3k\,\omega_{k,i}\left({}_f\langle 0|\alpha^{r\dagger}_{\vec{k},i}\alpha^{r}_{\vec{k},i}|0\rangle_f +_f \langle 0|\beta^{r\dagger}_{\vec{k},i}\beta^{r}_{\vec{k},i}|0\rangle_f\right) \\
&= 8\sin^2\theta \int_0^K d^3k(\omega_{k,1}+\omega_{k,2})|V_{\vec{k}}|^2 .
\end{aligned}
$$

where $\eta_{00} = 1$ in a Robertson-Walker (cosmological) metric background.

Consistent choice of cutoff scale, $K \equiv k_0 = m_1 + m_2$ (Barenboim + NM 2004) compatible with our decoherence-induced mass difference scenario.

For hierarchical neutrino models, i.e. $m_1 \gg m_2 \to k_0 \gg \sqrt{m_1 m_2}$, modes near the cutoff contribute most to the vacuum energy (divergence),

$$
\Lambda \equiv \langle \rho_{\rm vac}^{\nu-{\rm mix}}\rangle \sim 8\pi \sin^2\theta (m_1-m_2)^2(m_1+m_2)^2 \times \left(\sqrt{2}+1+\mathcal{O}(\frac{m_2^2}{m_1^2})\right) \propto \sin^2\theta(\Delta m^2)^2
$$

Matter-Antimatter Asymmetry and Decoherence

Sphaleron transitions occurring at and after the electroweak phase transition induce violations of $B+L$, which efficiently wipe out any pre-existing $B+L$ asymmetry. Leptogenesis models evade this problem by generating an early asymmetry in L, which is then converted to a baryon asymmetry by the $B-L$ conserving sphaleron processes.

To avoid sphaleron dilution of $B+L$, and to satisfy the Sakharov conditions for baryogenesis, standard leptogenesis models require strongly out-of-equilibrium processes and new sources of CP violation beyond the Standard Model.

Our model of decoherence on the contrary provides a novel and extremely economical mechanism to generate the observed baryon asymmetry, through a process of equilibrium electroweak leptogenesis (the fact that it violates CPT obviates the need for two of the three Sakharov conditions, namely the requirements of out-of-equilibrium and CP violating processes).

By CPTV we have violations of the index theorem that relates the Chern-Simons winding number of the sphaleron configuration to a change in $B+L$.

"Medium-induced Decoherence" & Neutrinos

NEUTRINO OSCILLATIONS IN (NOISY) MEDIA

NEUTRINO PROPAGATION IN A MEDIUM WITH, SAY, ELECTRON DENSITY n_e (e.g. the Sun environment) (Mikheyev-Smirnov (1986), Wolfenstein (1978))

MSW EFFECT

MASS-SQUARED DIFFERENCE (and other effects, e.g. spin precession) BETWEEN ν FLAVOURS IS DEVELOPED AS A RESULT OF THE PASSAGE OF ν THROUGH MATTER, EVEN IF ν WERE DEGENERATE IN MASS IN VACUO.

Mixing angle: $\sin^2 2\tilde{\theta} = \sin^2 2\theta \left(\frac{\Delta m^2}{\Delta \tilde{m}^2}\right)$

Mass-Squared Difference:
$\Delta \tilde{m}^2 = \sqrt{(D - \Delta m^2 \cos 2\theta)^2 + (\Delta m^2 \sin 2\theta)^2}$

Tilde= Medium quantity, Untilde= vacuum quantity.

$D = \sqrt{2} G_F n_e k$, (G_F =Fermi's const, k =momentum scale)

PHYSICALLY: Charged current interact only with ν_e:
$\frac{G_F}{\sqrt{2}} \overline{\nu_e} \gamma_\lambda (1 + \gamma_5) e \overline{e} \gamma^\lambda (1 + \gamma_5) \nu_e$.

stochastic MSW & Decoherence

MSW EFFECT CAN BE GENERALISED TO STOCHASTICALLY FLUCTUATING MEDIA (Loreti, Balantekin (1994))

Fluctuating (in time) medium electron density

$$\langle n_e \rangle = n_{e,0} \equiv n_0$$
$$\langle n_e(t) n_e(t') \rangle = n_0^2 \Omega^2 \delta(t - t') + \text{higher correlations}$$

We set from now on $t = r$ $(c = 1)$.

Temporal evolution of density matrix of matter system (ν in our example) $\rho = \mathrm{Tr}|\psi\rangle\langle\psi| \equiv \psi \otimes \psi^\dagger$ Tr =unobserved degrees of freedom.

If ψ obeys Schrödinger eq. $i\frac{d}{dt}\psi = \hat{H}\psi$,
$\psi^T(t) = (\psi_1, \psi_2, \ldots \psi_N)$ for N-level system, then:

$$i\frac{d\hat{\rho}}{dt} = [\hat{H}, \hat{\rho}]; \quad \hat{H} = \hat{H}_0 + B(t)\hat{M}'$$

H_0 mean field effects, $\hat{M}'$ independent of time, $B(t)$ fluctuating field, $\langle B(t) \rangle = 0$, $\langle B(t_1)B(t_2)\rangle = \alpha^2 f(|t_1 - t_2|)$.

GAUSSIAN FIELD: $\langle B(t_1)B(t_2)\rangle = 2\tau\alpha^2\delta(t_1 - t_2)\ldots$

$$\frac{d}{dt}\langle\hat{\rho}(t)\rangle = -i[\hat{H}_0, \langle\hat{\rho}(t)\rangle] - \alpha^2\tau[\hat{M}', [\hat{M}', \langle\hat{\rho}(t)\rangle]]$$

NB: DOUBLE COMMUTATOR IS TIME IRREVERSIBLE, unrelated to CP $\rightarrow$ CPT Violating due to matter.

Space-time foam induced MSW?

MSW effects: neutrinos in matter $\Delta m^2 \sim G_F n_e k$,

n_e electronic density of medium, G_F Fermi's (weak interaction) constant, k momentum scale of neutrino Medium discriminates between flavour: only ν_e interact with charged currents.

Idea (Barenboim + NM (2004)): what about QG foam effects? charged black hole antiblack hole pairs can create by Hawking radiation and Hawking absorption (CPT mirror process for antiblack holes) local fluctuations in the density of charged foam particles in the medium. Stochastic fluctuations of these charge densities due to back reaction effects on foam (metric fluctuations).

Semi-classical computations (Zhang et al., Lifschytz) show that if such black holes are near-extremal then their Hawking radiation rate is lower than their neutral counterparts.

Evaporation of such black holes create preferentially $e^- e^+$, because they are lighter, leading to vacuum electron charge fluctuations. ν interact with the foam, due to standard model ν interactions, then, one has an MSW-like effect, but gravitational in origin. *Gravitationally-induced ν-mass differences ?*

Space-time foam induced MSW?

MSW in stochastic media already studied in detail (Loreti & Balantekin 1994, Benatti & Floreanini 2005).

In our model, Neutrinos interact with such charge densities of particles emitted by the QG foam, assume stochastic Gaussian medium with density fluctuations about a mean value $n_0(k) \propto k^{-1}$: the higher the momentum the lesser the number of foam particles the ν $(\overline{\nu})$ interacts with.

Gravitational MSW effect:

$$(\Delta m^2)_{\rm foam} \sim G_N n_0 k \ ,$$

$n_0(k)k \sim k$-independent, $G_N \sim 1/M_P^2$.

Stochastic QG fluctuations origin of Δm^2 COMMON in both sectors (average n_0 same in BOTH sectors)

Space-time foam induced MSW?

We take our space-time foam interaction Hamiltonian, H_I, (for the two generation case for definiteness) to be of the general diagonal form in flavour space

$$H_I = \begin{pmatrix} a_{\nu_e} & 0 \\ 0 & a_{\nu_\mu} \end{pmatrix} \qquad (1)$$

where we expect

$$a_{\nu_e} - a_{\nu_\mu} \propto G_N \langle n^c_{\rm bh}(r) \rangle k$$

NB: for $k \sim m_1 + m_2$ (dominant modes for Λ) the induced foam mass splittings $\Delta m^2_{\rm foam} \sim G_N \langle n^c_{\rm bh} \rangle (m_1 + m_2)$ from which $m_1 - m_2 \sim \langle n^c_{\rm bh} \rangle / M_P^2$. If we assume there are $\mathcal{N}_c$ charged foam-induced objects per Planck volume, $V_P \sim M_P^{-3}$ then,

$$\mathcal{N}_{c,\rm max} \sim m_1 - m_2 / M_P$$

which is small, consistent with mathematical properties of charged black/white hole pairs in foam.

Foam-Density Fluctuations and CPTV

But there are fluctuations (Gaussian) of $n(r)$:

$\langle n^c_{\rm bh}(r) n^c_{\rm bh}(r')\rangle \sim \Omega^2 n_0^2 \delta(r - r')$, $< n^c_{\rm bh}(r) >= n_0$,

Effective neutrino Hamiltonian will assume the generic form $H_{\rm eff} = H + n^c_{\rm bh}(r) H_I$, where $H_I = G_N J_{fxf}$, is an appropriate constant $f \times f$ matrix, whose entries depend on the details of the foam/neutrino interactions

Evolution of neutrino density matrix in such media:

$$\partial_t \langle \rho \rangle = i[H + n_0 H_I, \langle \rho \rangle] - \Omega^2 n_0^2 [H_I, [H_I, \langle \rho \rangle]] =$$
$$i H^- \langle \rho \rangle - i \langle \rho \rangle H^+ + 2\Omega^2 n_0^2 H_I \langle \rho \rangle H_I,$$

where $H^{\pm} = H_{\rm eff} \pm i \Omega^2 n_0^2 H_I^2$, and $\langle \dots \rangle$ indicates average with respect to the stochastic effects.

The Hamiltonian part: space-time foam-induced mass-squared MSW-like splittings for neutrinos (mean field). The double commutator fluctuation decoherence part: is *time irreversible*, unrelated in principle to CP properties, and thus **CPT violating**. Similar to energy-driven decoherence models (Houghston 1996, Adler 2000)

Due to CPTV one may have:

$$\overline{\Omega} \neq \Omega$$

while $\langle n^c_{\rm bh}\rangle \equiv n_0$ the same in both sectors.

FOAM DECOHERENCE: FORMALISM

Major approaches:

(i) Lindblad (linear) model-independent formalism (not specific to foam):

Requirements: (i) Energy conservation on average, (ii)(complete) positivity of ρ, (iii) monotonic entropy increase

Generic Decohering Lindblad Evolution:

$$\frac{\partial \rho_\mu}{\partial t} = \sum_{ij} h_i \rho_j f_{ij\mu} + \sum_\nu L_{\mu\nu} \rho_\mu \ ,$$
$$\mu, \nu = 0, \ldots N^2 - 1, \quad i, j = 1, \ldots N^2 - 1 \qquad (2)$$

for N-level systems, where h_i Hamiltonian terms.

Example for three generation neutrino oscillations: $N = 3$, f_{ijk} structure constants of SU(3).

Entropy increase requirement:

$$L_{0\mu} = L_{\mu 0} = 0 \ ,$$

$$L_{ij} = \frac{1}{4} \sum_{k,\ell,m} c_{l\ell} \left(-f_{i\ell m} f_{kmj} + f_{kim} f_{\ell mj} \right) \ ,$$

with c_{ij} a positive definite matrix (non-negative eigenvalues).

FOAM-MSW & LINDBLAD DECOHERENCE

Double commutator structures in stochastic MSW evolution assume a Lindblad form.

In the stochastic MSW case the c-matrix is

$$c_{kl} = \Omega^2 \begin{pmatrix} {h'_1}^2 & 0 & h'_1 h'_3 \\ 0 & 0 & 0 \\ h'_1 h'_3 & 0 & {h'_3}^2 \end{pmatrix}$$

where $h_1 = (a_{\nu_e} - a_{\nu_\mu})\sin(2\theta)$, $h_3 = (a_{\nu_e} - a_{\nu_\mu})\cos(2\theta)$, $a_{\nu_e} - a_{\nu_\mu} \propto n_0 G_N k$.

We can easily see that complete positivity is guaranteed.

And we obtain the probability for oscillations (for two generations)

$$\begin{aligned}
P_{\nu_e \to \nu_\mu} &= \mathrm{Tr}\rho_{\nu_\mu}(t)\rho_{\nu_e} = \sin(\Phi) e^{-\frac{t\Omega^2}{4}(h_1^2 - h_3^2)} \sin^2(2\theta)\frac{\Omega^2(2h_3^2 - h_1^2)}{4\Delta_{21}} \\
&+ \cos(\Phi) e^{-\frac{t\Omega^2}{4}(h_1^2 - h_3^2)} \Bigg(-\frac{h_1^2}{2\Delta_{21}^2}\cos^2(2\theta) \\
&+ \sin^2(2\theta)\left(\frac{h_3^2}{2\Delta_{21}^2} - \frac{1}{2}\right) + \frac{h_1 \sin(4\theta)}{2\Delta_{21}} \Bigg) \\
&+ e^{\frac{-t\Omega^2 h_1^2}{2}} \Big(2h_1^2\cos(4\theta) + (2h_1 h_3 - 2h_1\Delta_{21})\sin(4\theta) \\
&\quad + \frac{1}{2}\sin^2(2\theta) \Big)
\end{aligned}$$

$$\Phi = t(h_3 + \frac{h_1^2}{\Delta_{21}} + \Delta_{21}), \qquad \Delta_{21} = \frac{\Delta m^2}{2p}$$

NB: Note Lindblad $e^{-(\ldots)t}$ suppression

3-generation Lindblad Oscillation Probability

(Barenboim, NM, Sarben Sarkar, Waldron 2006)

$$P_{\nu_\alpha \to \nu_\beta}(t) = \mathrm{Tr}(\rho_{\nu_\beta}(t)\rho_{\nu_\alpha}) = \frac{1}{3} +$$

$$\frac{1}{2}\left\{\left[\rho_1^\alpha \rho_1^\beta \cos\left(\frac{|\Omega_{12}|t}{2}\right) + \left(\frac{\Delta L_{21}\rho_1^\alpha \rho_1^\beta}{|\Omega_{12}|}\right)\sin\left(\frac{|\Omega_{12}|t}{2}\right)\right] e^{(L_{11}+L_{22})\frac{t}{2}}\right.$$

$$+ \quad \left[\rho_4^\alpha \rho_4^\beta \cos\left(\frac{|\Omega_{13}|t}{2}\right) + \left(\frac{\Delta L_{54}\rho_4^\alpha \rho_4^\beta}{|\Omega_{13}|}\right)\sin\left(\frac{|\Omega_{13}|t}{2}\right)\right] e^{(L_{44}+L_{55})\frac{t}{2}}$$

$$+ \quad \left[\rho_6^\alpha \rho_6^\beta \cos\left(\frac{|\Omega_{23}|t}{2}\right) + \left(\frac{\Delta L_{76}\rho_6^\alpha \rho_6^\beta}{|\Omega_{23}|}\right)\sin\left(\frac{|\Omega_{23}|t}{2}\right)\right] e^{(L_{66}+L_{77})\frac{t}{2}}$$

$$+ \quad \left[\left(\rho_3^\alpha \rho_3^\beta + \rho_8^\alpha \rho_8^\beta\right)\cosh\left(\frac{\Omega_{38}t}{2}\right)\right.$$

$$+ \quad \left.\left(\frac{2L_{38}(\rho_3^\alpha \rho_8^\beta - \rho_8^\alpha \rho_3^\beta) + \Delta L_{83}\left(\rho_3^\alpha \rho_3^\beta - \rho_8^\alpha \rho_8^\beta\right)}{\Omega_{38}}\right)\sinh\left(\frac{\Omega_{38}t}{2}\right)\right]$$

$$\left. e^{(L_{33}+L_{88})\frac{t}{2}}\right\}$$

$\Delta L_{ij} \equiv L_{ii} - L_{jj}$, $\Omega_{12} = \sqrt{(L_{22}-L_{11})^2 - 4\left(\frac{\Delta m_{12}^2}{2p}\right)^2}$, $\Omega_{13} = \sqrt{(L_{44}-L_{55})^2 - 4\left(\frac{\Delta m_{13}^2}{2p}\right)^2}$, $\Omega_{23} = \sqrt{(L_{66}-L_{77})^2 - 4\left(\frac{\Delta m_{23}^2}{2p}\right)^2}$, $\Omega_{38} = \sqrt{(L_{33}-L_{88})^2 + 4L_{38}^2}$,

$$\rho_0^\alpha = \sqrt{\frac{2}{3}},\ \rho_1^\alpha 2Re(U_{\alpha 1}^* U_{\alpha 2}),\ \rho_2^\alpha - 2Im(U_{\alpha 1}^* U_{\alpha 2}),\ \rho_3^\alpha |U_{\alpha 1}|^2 - |U_{\alpha 2}|^2,$$

$$\rho_4^\alpha 2Re(U_{\alpha 1}^* U_{\alpha 3}),\ \rho_5^\alpha - 2Im(U_{\alpha 1}^* U_{\alpha 3}),\ \rho_6^\alpha 2Re(U_{\alpha 2}^* U_{\alpha 3}),$$

$$\rho_7^\alpha - 2Im(U_{\alpha 2}^* U_{\alpha 3}),\ \rho_8^\alpha \sqrt{\frac{1}{3}}\left(|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 - 2|U_{\alpha 3}|^2\right)$$

NB: Note the Lindblad $e^{-(\ldots)t}$ suppression

FOAM DECOHERENCE: FORMALISM

BEYOND LINDBLAD

(ii) Non-critical Strings (possibly non-linear, specific to QG foam) (**Ellis, NM, Nanopoulos 1992)**:

$$\partial_t \rho = i[\rho, H] + :\beta^i < V_i V_j > [g^j, \rho]:,$$

where $< ... >$ hides non linearities, $g^i = g_{\mu\nu}, ...$ string backgrounds, $\beta^i = \sum_n C^i_{i_1 \dots i_n} g^{i_1} \dots g^{i_n}$, describes deviation from conformal invariance on the world sheet (foam effect). **Can include Lindblad as a special case**

(iii) Fokker-Planck equation for probability density P distributions with diffusion $\mathcal{D}$,

$$\partial_t P = \mathcal{D} \nabla^2 P + \nabla \cdot \mathcal{J}$$

diffeomorphism invariant, leading to non-linear Schrödinger equation (Doebner-Goldin) for matter wavefunction ψ in gravitational environment (no use of density matrices):

$$i\hbar \partial_t \Psi = -\frac{\hbar^2}{2m} \nabla^2 \Psi + i\mathcal{D}\hbar \left(\nabla^2 \Psi + \frac{|\nabla \Psi|2}{|\Psi|^2} \Psi \right)$$

if foam-induced diffusion: $\mathcal{D} = O\left((E/M_P)^n\right)$.

BUT **supersymmetry** implies **linearity** in string-inspired models **(NM & Szabo 2001, NM 2004)**.

FOAM DECOHERENCE: FORMALISM

BEYOND LINDBLAD II

(iv) Stochastically fluctuating space times with metrics fluctuating along direction of motion (for simplicity) **(Sarben Sarkar, A. Waldron, NM)**

$$g^{\mu\nu} = \begin{pmatrix} -(a_1+1)^2 + a_2^2 & -a_3(a_1+1) + a_2(a_4+1) \\ -a_3(a_1+1) + a_2(a_4+1) & -a_3^2 + (a_4+1)^2 \end{pmatrix}.$$

with random variables $\langle a_i \rangle = 0$ and $\langle a_i a_j \rangle = \delta_{ij}\sigma_i$.

EXAMPLE: Two generation Dirac neutrinos with MSW interaction V (of unspecified origin, could be space-time foam effect) oscillation probability:

$$\langle e^{i(\omega_1-\omega_2)t}\rangle = e^{i\frac{\left(z_0^+ - z_0^-\right)t}{k}} e^{-\frac{1}{2}\left(-i\sigma_1 t\left(\frac{(m_1^2-m_2^2)}{k}+V\cos 2\theta\right)\right)} \times$$
$$e^{-\frac{1}{2}\left(\frac{i\sigma_2 t}{2}\left(\frac{(m_1^2-m_2^2)}{k}+V\cos 2\theta\right)-\frac{i\sigma_3 t}{2}V\cos 2\theta\right)} \times$$
$$e^{-\left(\frac{(m_1^2-m_2^2)^2}{2k^2}(9\sigma_1+\sigma_2+\sigma_3+\sigma_4)+\frac{2V\cos 2\theta(m_1^2-m_2^2)}{k}(12\sigma_1+2\sigma_2-2\sigma_3)\right)t^2}$$

where $\Upsilon = \frac{Vk}{m_1^2-m_2^2}$, $|\Upsilon| \ll 1$, and $k^2 \gg m_1^2, m_2^2$, and

$$z_0^+ = m_1^2 + \Upsilon(1+\cos 2\theta)(m_1^2-m_2^2) + \Upsilon^2(m_1^2-m_2^2)\sin^2 2\theta$$
$$z_0^- = m_2^2 + \Upsilon(1-\cos 2\theta)(m_1^2-m_2^2) - \Upsilon^2(m_1^2-m_2^2)\sin^2 2\theta.$$

NB: σ-modifications of oscil. period, $e^{-(\ldots)t^2}$ suppression.

Uncertainty induced Decoherence

Gaussian Averaged ν-oscillations can produced Decoherence (T. Ohlsson, hep-ph/0012272)

Recall oscillation formula:

$$P_{\alpha\beta} = P_{\alpha\beta}(L,E) =$$
$$\delta_{\alpha\beta} - 4\sum_{a=1}^{n}\sum_{\beta=1,a<b}^{n} \mathrm{Re}\left(U^*_{\alpha a}U_{\beta a}U_{\alpha b}U^*_{\beta b}\right)\sin^2\left(\frac{\Delta m^2_{ab}L}{4E}\right) -$$
$$2\sum_{a=1}^{n}\sum_{b=1,a<b}^{n} \mathrm{Im}\left(U^*_{\alpha a}U_{\beta a}U_{\alpha b}U^*_{\beta b}\right)\sin\left(\frac{\Delta m^2_{ab}L}{2E}\right)$$

where $\alpha, \beta = e, \mu, \tau, ...,\ a, b = 1, 2, ...n$, $\Delta m^2_{ab} = m^2_a - m^2_b$

BUT...UNCERTAINTIES for E IN PRODUCTION OF nu-WAVE; Also: NOT WELL-DEFINED PROPAGATION LENGTH L :

$$\Delta E \neq 0, \qquad \Delta L \neq 0$$

Hence, have to AVERAGE Oscillation Probability P over L/E Dependence.

Gaussian Average Decoherence

GAUSSIAN AVERAGE: Approximate $\langle L/E \rangle \simeq \langle L \rangle / \langle E \rangle$

$$\langle P \rangle = \int_{-\infty}^{\infty} \mathrm{d}x \; P(x) \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\ell)^2}{2\sigma^2}}$$

$\ell \equiv \langle x \rangle$, $\sigma = \sqrt{\langle (x - \langle x \rangle)^2}$, $x = L/4E$.

AVERAGE $\langle P_{\alpha\beta} \rangle$:

$$\langle P_{\alpha\beta} \rangle = \delta_{\alpha\beta} -$$
$$2 \sum_{a=1}^{n} \sum_{\beta=1, a<b}^{n} \mathrm{Re}\left(U^*_{\alpha a} U_{\beta a} U_{\alpha b} U^*_{\beta b}\right) \left(1 - \cos(2\ell\Delta m^2_{ab}) e^{-2\sigma^2(\Delta m^2_{ab})^2}\right)$$
$$-2 \sum_{a=1}^{n} \sum_{b=1, a<b}^{n} \mathrm{Im}\left(U^*_{\alpha a} U_{\beta a} U_{\alpha b} U^*_{\beta b}\right) \sin(2\ell\Delta m^2_{ab}) e^{-2\sigma^2(\Delta m^2_{ab})^2}$$

NB: Damping factors due to σ (!)

EXAMPLE: TWO FLAVOURS

$$\langle P_{\alpha\beta} \rangle = \frac{1}{2}\sin^2 2\theta \left(1 - e^{-2\sigma^2(\Delta m^2)^2}\cos(2\ell\Delta m^2)\right), \; \ell = \frac{\langle L \rangle}{4\langle E \rangle}$$

Bounds on σ (T. Ohlsson)

- Pessimistic: $\sigma \simeq \Delta x \simeq \Delta \frac{L}{4E} \leq \frac{\langle L \rangle}{4\langle E \rangle}\left(\frac{\Delta L}{\langle L \rangle} + \frac{\Delta E}{\langle E \rangle}\right)$
- Optimistic: $\sigma \leq \frac{\langle L \rangle}{4\langle E \rangle}\left([\frac{\Delta L}{\langle L \rangle}]^2 + [\frac{\Delta E}{\langle E \rangle}]^2\right)^{1/2}$

Equivalence with decoherence:

Lindblad: $\dot{\rho} = i[\rho, H] + \mathcal{D}[\rho]$,

$$\mathcal{D}[\rho] = \sum_{i=1}^{n} [D_i, [D_i, \rho]]$$

(if $D_i^\dagger = D_i$, energy is conserved on average, and the ρ is a completely positive map) (Adler 2000)

Example: TWO FLAVOURS: One Decoherence Coefficient γ:

$$P_{e\mu}(L, E) = \frac{1}{2}\sin^2 2\theta \left(1 - e^{-\gamma L}\cos(\frac{\Delta m^2 L}{2E})\right)$$

($L = t$, $c = 1$).

COMPARE WITH "FAKE" GAUSSIAN AVERAGE:

$$2\sigma^2(\Delta m^2)^2 = \gamma L \quad \rightarrow \quad \gamma = \frac{(\Delta m^2)^2}{8E^2} L r^2$$

with $\sigma = (L/4E)r$, $r = \frac{\Delta L}{L} + \frac{\Delta E}{E}$ (pessimistic), or $r = \sqrt{(\frac{\Delta L}{L})^2 + (\frac{\Delta E}{E})^2}$ (optimistic).

For atmospheric ν: $\sigma_{\rm atm} \sim 1.5 \times 10^3$ eV^2 (for $L \sim 12000\ Km$), $r \sim \mathcal{O}(1)$, hence

$$\gamma_{\rm atm, fake} < 10^{-24}\ {\rm GeV}$$

COMPARE WITH QG: (i) optimistic (Ellis, NM, Nanopoulos) : $\gamma_{QG} \sim E^2/M_{QG}$, (ii) pessimistic: (Adler) $\gamma_{QG} \sim \frac{(\Delta m^2)^2}{E^2 M_{QG}}$.

NB: In QG NO L Dependence, but $1/M_{QG}$ (in 4-dim $M_{QG} \sim M_P \sim 10^{19}$ GeV) CAN DISENTANGLE (!)

Quantum Gravity Uncertainties

NB: GAUSSIAN AVERAGE ALSO DUE TO QUANTUM-GRAVITY UNCERTAINTIES:

If Δ/L is due to "Fuzziness" of space time due to quantum fluctuations, then (Van Dam, Ng, Ellis, NM, Nanopoulos)

$$\frac{\Delta L}{L}, \quad \frac{\Delta E}{E} \sim \beta \left(\frac{E}{M_{QG}}\right)^{\alpha},$$

α some positive integer, $\alpha \geq 1$, $\beta = \beta(L)$ some coefficient. In this case $r \sim \beta \left(\frac{E}{M_{QG}}\right)^{\alpha}$.

Then, from Gaussian Average we get for Decoherence:

$$\gamma \sim \frac{(\Delta m^2)^2}{8E^2} \beta \left(\frac{E}{M_{QG}}\right)^{\alpha} L$$

NB: modified E-dependence, but still $\propto L$ if β=const.

INTERESTING TO EXPLORE FURTHER...(c.f. below)

HOWEVER, IN GENERAL SUCH EFFECTS CAN BE DISENTANGLED FROM OTHER α, β, γ COEFFICIENTS OR STOCHASTIC-MEDIUM EFFECTS BY THEIR L DEPENDENCE...

Genuine vs “Fake” CPTV & Decoherence Effects

Important to distinguish: Intrinsic (genuine, due to QG) from Extrinsic (“fake”) CPTV effects due to matter influences.

SOME NOMENCLATURE

Probability differences:
$P_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta), P_{\overline{\alpha}\overline{\beta}} = P(\overline{\nu_\alpha} \to \overline{\nu_\beta})$, Greek indices=flavour.

- (I) CP: $\Delta P^{\rm CP}_{\alpha\beta} = P_{\alpha\beta} - P_{\overline{\alpha}\overline{\beta}}$
- (II) T: $\Delta P^{\rm T}_{\alpha\beta} = P_{\alpha\beta} - P_{\beta\alpha}$
- (III) CPT: $\Delta P^{\rm CPT}_{\alpha\beta} = P_{\alpha\beta} - P_{\overline{\beta}\overline{\alpha}}$

Probability Conservation for ‘fake’ CPTV:
$\sum_{\alpha=e,\mu,\tau,\ldots} \Delta P^{\rm CPT}_{\alpha\beta} = \sum_{\beta=e,\mu,\tau,\ldots} \Delta P^{\rm CPT}_{\alpha\beta} = 0$ and $\Delta P^{\rm CPT}_{\alpha\beta} = -\Delta P^{\rm CPT}_{\overline{\beta}\overline{\alpha}}$ i.e. probability difference for $\overline{\nu}$ do not give further information. CONTRAST WITH GENUINE CPTV where $\Delta P^{\rm CPT}_{\alpha\beta} \neq \Delta P^{\rm CPT}_{\overline{\beta}\overline{\alpha}}$ due to different decoherence parameters between ν and $\overline{\nu}$ sectors.

L/E dependence of $\Delta P^{\rm CPT}$ due to matter would distinguish it from QG effects, where one might have enhancement with ν energy E (c.f. below).

Order of "Fake" CPTV

Experiment	CPT probability differences	
	Quantities	Numerical value
BNL NWG	$\Delta P_{\mu e}^{\mathrm{CPT}}$	0.010
BNL NWG	$\Delta P_{\mu e}^{\mathrm{CPT}}$	0.032
BooNE	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$6.6 \cdot 10^{-13}$
MiniBooNE	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$4.1 \cdot 10^{-14}$
CHOOZ	$\Delta P_{ee}^{\mathrm{CPT}}$	$-3.6 \cdot 10^{-5}$
ICARUS	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$4.0 \cdot 10^{-5}$
	$\Delta P_{\mu \tau}^{\mathrm{CPT}}$	$-3.8 \cdot 10^{-5}$
JHF-Kamioka	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$3.8 \cdot 10^{-3}$
	$\Delta P_{\mu \mu}^{\mathrm{CPT}}$	$-1.3 \cdot 10^{-4}$
K2K	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$1.0 \cdot 10^{-3}$
	$\Delta P_{\mu \mu}^{\mathrm{CPT}}$	$-5.3 \cdot 10^{-5}$

Experiment	CPT probability differences	
	Quantities	Numerical value
KamLAND	$\Delta P_{ee}^{\mathrm{CPT}}$	-0.033
LSND	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$4.8 \cdot 10^{-15}$
MINOS	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$1.9 \cdot 10^{-4}$
	$\Delta P_{\mu \mu}^{\mathrm{CPT}}$	$-1.1 \cdot 10^{-5}$
NuMI I	$\Delta P_{\mu e}^{\mathrm{CPT}}$	0.026
NuMI II	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$2.6 \cdot 10^{-3}$
NuTeV	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$1.6 \cdot 10^{-18}$
NuTeV	$\Delta P_{\mu e}^{\mathrm{CPT}}$	$8.2 \cdot 10^{-20}$
OPERA	$\Delta P_{\mu \tau}^{\mathrm{CPT}}$	$-3.8 \cdot 10^{-5}$
Palo Verde	$\Delta P_{ee}^{\mathrm{CPT}}$	$-1.2 \cdot 10^{-5}$
Palo Verde	$\Delta P_{ee}^{\mathrm{CPT}}$	$-2.2 \cdot 10^{-5}$

Table 1: Extrinsic CPT pds for some past, present, and future long-baseline experiments (Jacobson-Ohlsson, hep-ph/0305064).

NB: Extrinsic CPTV negligible for future ν factories ($\sim 10^{-5}$), sensitive to genuine CPTV? (study for 2 cases: $L \sim 3000\ Km, 7000\ Km, hep-ph/0305064$)

FITTING ν DATA with genuine CPTV Decoherence?

In progress: Barenboim, Sarben Sarkar, Waldron, NM

Using CPTV asymmetry between ν and $\overline{\nu}$ sectors, we assume decoherence only in $\overline{\nu}$ sector.

We managed to fit the 3-generation Lindblad probabilities preserving positivity and boundedness, with ALL data, including LSND and KamLand.

To fit spectral distortion seen by KamLand need to impose general conditions: $L_{11} = L_{22}$, $L_{44} = L_{55}$,

$L_{66} = L_{77}$, $L_{33} = L_{88}, L_{38} = L_{83} = 0$

Then, imposing the special conditions

$L_{33} = L_{66} = 0$, $L_{11} = L_{22} = L_{44} = L_{55} = -|A| \propto -\frac{1}{L} < 0$

we obtain excellent fits to the data.

Positivity: The solutions for c_{ij} is such that the only non-zero elements are $c_{88} = \frac{2c_{38}}{\sqrt{(3)}}$ (note $c_{38} = c_{83}$). This leads to non-negative eigenvalues $(0,0,0,0,0,0,0,2/\sqrt{(3)})$.

The $1/L$-behaviour in decoherence Lindblad exponents points towards stochastic fluctuations (c.f. above) of length, of the form:

$$(\Delta L)^2 \propto E^2/M_P^4$$

compatible with QG expected behaviour.

CONCLUSIONS

Neutrino Physics may provide a very useful guide in our quest for a theory of Quantum Gravity, in particular stringent constraints on CPT Violation. The latter may not be an academic issue, but a real feature of QG.

Quantum Gravity may exhibit Lorentz Invariant (and hence frame independent) CPTV Decoherence. Theoretically the presence of an environment may be consistent with Lorentz Invariance (Millburn 2003). The scenario of three-generation antineutrino decoherence + mixing is still compatible with all the existing ν data, including LSND and KamLAND, and also yields interesting estimates for Dark Energy and matter-antimatter asymmetry, compatible with known estimates and limits, observations.

A peculiar relation between Δm^2, Dark Energy, and QG foam has been proposed, which stills appears compatible with the data: QG-foam induced MSW effect?

More work (Theory & Expt) to be done before conclusions are reached...