$0\nu\beta\beta$-decay; the problem of NME

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The observation of neutrino oscillations in SK, SNO, KamLAND, K2K, Homestake, SAGE, GALLEX-GNO means

$$\nu_{lL} = \sum_i U_{li} \nu_{iL}$$

U is PMNS mixing matrix

ν_i is the field of neutrino with mass m_i

Dirac or Majorana?

The answer to this fundamental question will have enormous impact on physics of massive and mixed neutrinos

If ν_i are Majorana, it will be proof that neutrino masses and mixing are due to a new physics

Strong argument in favor of the the most plausible see-saw mechanism of neutrino mass generation

Investigation of neutrino oscillations can not reveal the nature of ν_i

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = |\sum_i U_{\alpha' i}\ e^{-i\Delta m^2_{i1}\frac{L}{2E}}\ U^*_{\alpha i}|^2$$

$$U^{Mj} = U^D S(\beta)$$

$S(\beta)$ is a diagonal phase matrix

$$\mathrm{P}^{Mj}(\nu_\alpha \to \nu_{\alpha'}) = \mathrm{P}^D(\nu_\alpha \to \nu_{\alpha'})$$

No difference between D and Mj in evolution equation of neutrino in matter

The most sensitive method to reveal the nature of ν_i is the search for $0\nu\beta\beta$-decay

$$(\mathrm{A}, \mathrm{Z}) \to (\mathrm{A}, \mathrm{Z}+2) + \mathrm{e}^- + \mathrm{e}^-$$

of ^{76}Ge, ^{130}Te, ^{136}Xe, ^{100}Mo and other even-even nuclei for which usual β-decay is forbidden

The standard interaction Hamiltonian

$$\mathcal{H}_I^{CC} = \frac{G_F}{\sqrt{2}}\, 2\bar{e}_L \gamma_\alpha \nu_{eL}\ j^\alpha + \text{h.c.}$$

Majorana neutrino mixing

$$\nu_{eL} = \sum_i U_{ei}\, \nu_{iL}; \quad \nu_i = \nu_i^c = C\, \bar{\nu}_i^T$$

$0\nu\beta\beta$ is the second order in the Fermi constant process with virtual neutrinos

Neutrino properties in the neutrino propagator

For small neutrino masses

$$\langle 0|T(\nu_{eL}(x_1)\nu_{eL}^T(x_2)|0\rangle \simeq$$
$$m_{ee}\ \frac{i}{(2\,\pi)^4} \int e^{-ip(x_1-x_2)}\, \frac{1}{p^2} d^4p\, \frac{1-\gamma_5}{2}\, C$$

The $0\nu\beta\beta$ matrix element is proportional to the effective Majorana mass

$$\boxed{m_{ee} = \sum_i U_{ei}^2\, m_i}$$

The half-life

$$\frac{1}{T^{0\nu}_{1/2}(A,Z)} = |m_{ee}|^2 \, |M^{0\nu}(A,Z)|^2 \, G^{0\nu}(E_0,Z)$$

$G^{0\nu}(E_0,Z)$ **known phase-space factor**

$M^{0\nu}(A,Z)$ **nuclear matrix element**

NME does not depend on neutrino masses

includes propagator of massless neutrinos

many intermediate nuclear states must be taken into account

The results of many experiments on the search for $0\nu\beta\beta$-decay are available

Heidelberg-Moscow

^{76}Ge crystals, 86% enreached (11 kg)

$T^{0\nu}_{1/2} \geq 1.9 \cdot 10^{25}$ years; $|m_{ee}| \leq (0.3 - 1.2)$ eV

CUORICINO

^{130}Te, cryogenic detector (40.7 kg of TeO_2)

$T^{0\nu}_{1/2} \geq 7.5 \cdot 10^{23}$ years; $|m_{ee}| \leq (0.3 - 1.7)$ eV

ranges in $|m_{ee}|$ because of NME uncertainties

Future goal $|m_{ee}| \simeq$ a few 10^{-2}eV

Many projects

CUORE

Cryogenic detector, 800 kg $Te\,O_2$ crystals, resolution 5 keV

$T^{0\nu}_{1/2} \simeq 9.5 \cdot 10^{26}$ years; $|\mathrm{m_{ee}}| \simeq (2.0 - 5.2)\,10^{-2}$ eV

EXO

60-80 % enreached ^{136}Xe , $\simeq$ 10 tons; Ba^{+} atoms from $^{136}Xe \to ^{136} Ba^{++} + e^{-} + e^{-}$ will be optically tagged with lasers

$T^{0\nu}_{1/2} \simeq 1 \cdot 10^{28}$ years; $|\mathrm{m_{ee}}| \simeq (1.3 - 3.7)\,10^{-2}$ eV

GENIUS

1 ton 86 % enreached ^{76}Ge in liquid nitrogen

$T^{0\nu}_{1/2} \simeq 1 \cdot 10^{28}$ years; $|\mathrm{m_{ee}}| \simeq (1.3 - 5.0)\,10^{-2}$ eV

MAJORANA

500 kg of 86 % enreached ^{76}Ge

$T^{0\nu}_{1/2} \simeq 4 \cdot 10^{27}$ years; $|\mathrm{m_{ee}}| \simeq (2.1 - 7.0)\,10^{-2}$ eV

Effective Majorana mass $|m_{ee}|$ from neutrino oscillation data

Existing data (except LSND) are well described by three-neutrino mixing

To calculate

$$m_{ee} = \sum_{i=1}^{3} U_{ei}^2 \, m_i$$

we need to know m_i; U_{ei}^2

In the standard parametrization

$$U_{e1}^2 = \cos^2\theta_{13}\,\cos^2\theta_{12}\,e^{2\,i\alpha_1}$$

$$U_{e2}^2 = \cos^2\theta_{13}\,\sin^2\theta_{12}\,e^{2\,i\alpha_2};\ U_{e3}^2 = \sin\theta_{13}^2\,e^{2\,i\alpha_3}$$

α_i Majorana phases

From neutrino oscillation data we know

$$1.3\cdot 10^{-3} \le |\Delta m_{32}^2| \le 3.0\cdot 10^{-3}\mathrm{eV}^2$$

(Super Kamiokande, 90% CL)

$$\Delta m_{21}^2 = 7.9^{+0.6}_{-0.5}\cdot\cdot 10^{-5}\mathrm{eV}^2;\ \tan^2\theta_{12} = 0.40^{+0.10}_{-0.07}$$

(solar and KamLAND)

$$\sin^2\theta_{13} < 5\cdot 10^{-2}\ \text{(CHOOZ, 90\% CL)}$$

From Troitsk and Mainz tritium experiments

$$m_i \leq 2.3\,\mathrm{eV} \quad \mathrm{i} = 1, 2, 3$$

From cosmological data

$$\sum_i m_i \lesssim 1\,\mathrm{eV}$$

We do not know

- The pattern of neutrino mass spectrum (normal, inverted)
- Absolute values of neutrino masses
- Majorana phases

Three standard spectra

I. Hierarchy of masses

$$m_1 \ll m_2 \ll m_3$$

Neutrino masses are determined by the oscillation data

$$m_2 \simeq \sqrt{\Delta m_{21}^2} \simeq 8.9 \cdot 10^{-3}\mathrm{eV}.$$

$$m_3 \simeq \sqrt{|\Delta m_{32}^2|} \simeq 4.5 \cdot 10^{-2}\mathrm{eV}.$$

$$|m_{ee}| \simeq \left| \sin^2\theta_{12}\sqrt{\Delta m^2_{12}} + e^{i\,\alpha_{32}}\sin^2\theta_{13}\sqrt{\Delta m^2_{32}} \right|$$

The first term is small. Contribution of "large" $\sqrt{\Delta m^2_{32}}$ is suppressed by $\sin^2\theta_{13}$

Upper bound

$$|m_{ee}| \leq 4.6 \cdot 10^{-3}\text{eV}$$

significantly **smaller than the sensitivity of the future experiments**

II. Inverted hierarchy of masses

$$m_3 \ll m_1 < m_2$$

$$m_2 \simeq m_1 \simeq \sqrt{|\Delta m^2_{31}|};\ m_3 \ll \sqrt{|\Delta m^2_{31}|}$$

Effective Majorana mass

$$|m_{ee}| \simeq \sqrt{|\Delta m^2_{31}|}\ (1 - \sin^2 2\,\theta_{12}\sin^2\alpha_{21})^{\frac{1}{2}}$$

$$(1 - \sin^2 2\,\theta_{12})^{\frac{1}{2}}\sqrt{|\Delta m^2_{31}|} \leq |m_{ee}| \leq \sqrt{|\Delta m^2_{31}|}$$

Upper and lower bound correspond to CP conservation ($\eta_2 = \eta_3$ and $\eta_2 = -\eta_3$)

Lower bound is not equal to zero

From solar data $\sin^2 2\,\theta_{12} < 1(5.4\,\sigma)$

For the best-fit values of the parameters

$$0.8 \cdot 10^{-2}\mathrm{eV} \leq |\mathrm{m_{ee}}| \leq 5.5 \cdot 10^{-2}\mathrm{eV}$$

Can be reached in the future experiments

III. Practically degenerate neutrino masses

If $m_1 \gg \sqrt{\Delta m^2_{32}}$ ($m_3 \gg \sqrt{|\Delta m^2_{31}|}$)

$$m_1 \simeq m_2 \simeq m_3 \simeq m_0$$

Neglecting small contribution of $|U_{e3}|^3$

$$|m_{ee}| \simeq m_0 \ (1 - \sin^2 2\,\theta_{12}\,\sin^2\alpha)^{\frac{1}{2}}$$

The sensitivity of the future tritium KATRIN experiment

$$m_0 \simeq 0.2\,\mathrm{eV}$$

If positive effect will be find

$$0.42\,m_0 \leq |m_{ee}| \leq m_0$$

From the measurement $|m_{ee}|$ an information on the common mass m_0 can be obtained

$$|m_{ee}| \leq m_0 \leq 2.4\,|m_{ee}|$$

$|m_{ee}|$ for arbitrary $m_1(m_3)$ Fig.

If $0\nu\beta\beta$ is observed we will know that ν_i are Majorana particles

If $|m_{ee}|$ is known an important information about the character of neutrino mass spectrum, absolute values of neutrino masses and in principle Majorana CP phase will be obtained

However, from experimental data only product $|m_{ee}|^2 \, |M^{0\,\nu}(A,Z)|^2$ can be determined

Nuclear Matrix elements

Two standard methods of the calculations of NME : Quasiparticle random phase approximation and shell model

The results of different calculations of NME differ by factor 3 or more

For example at $|m_{ee}| = 5 \cdot 10^{-2}\mathrm{eV}$

$$6.8 \cdot 10^{26}\mathrm{y} \leq \mathrm{T}^{0\nu}_{1/2}(^{76}\mathrm{Ge}) \leq 70.8 \cdot 10^{26}\mathrm{y}$$

$$0.6 \cdot 10^{26}\mathrm{y} \leq \mathrm{T}^{0\nu}_{1/2}(^{130}\mathrm{Te}) \leq 23.2 \cdot 10^{26}\mathrm{y}$$

How to test nuclear matrix element calculations? (S.B. and S. Petcov)

Effective Majorana mass determined from decay of *different nuclei* must be the same

Relation between measured half-lifes

$$T^{0\nu}_{1/2}(A_2, Z_2) = X_M(A_2, Z_2 : A_1, Z_1)\ T^{0\nu}_{1/2}(A_1, Z_1) = ...$$

Coefficient $X_M(A_2, Z_2 : A_1, Z_1)$ is determined by ratio of $(NME)^2$

A model M is compatible with data if the relation holds

Does not mean that M gives correct $|m_{ee}|$

Three latest calculations

M_1 Shell Model E. Courier et al, 1999

M_2 QRPA V. Rodin et al, 2003 (g_{pp} from measured half- life of $2\nu\beta\beta$-decay)

M_3 QRPA O. Civitarese, J. Suhonen, 2003 (constants are fixed by β-decay of close nuclei)

	M_1	M_2	M_3
$X(^{100}\text{Mo};^{76}\text{Ge})$	—-	0.59	0.17
$X(^{130}\text{Te};^{76}\text{Ge})$	0.25	0.49	0.13
$X(^{136}\text{Xe};^{76}\text{Ge})$	0.55	0.80	0.07

The measurement of $0\nu\beta\beta$-decay of any of these three pairs of nuclei can tell us which model is compatible with data (if any)

Depends on the choice of nuclei

For ^{100}Mo and ^{130}Te

$$X(^{100}\text{Mo};^{130}\text{Te}) = 1.2 \ (\text{M}_2); \quad 1.3 \ (\text{M}_3)$$

If the relation between half-lives is fulfilled, both M_1 and M_2 are compatible with data

But $|m_{ee}|_{M_2} = 2.6 \ |m_{ee}|_{M_3}$

The observation of $0\nu\beta\beta$-decay of three (or more) nuclei could allow to determine $|m_{ee}|$ in a more model independent way

Conclusion

- The establishment of the nature of ν_i (Majorana or Dirac?) will have a profound importance for the understanding of the origin of small neutrino mass and neutrino mixing physics
- Far the most sensitive process is $0\nu\beta\beta$-decay
- Today's limit $|m_{ee}| \leq (0.3 - 1.2)$ eV
- Challenging goal of future experiments $|m_{ee}| \simeq$ a few 10^{-2}eV
- If $|m_{ee}|$ is measured the pattern of the neutrino mass spectrum will be revealed. possibly Majorana CP phase will be determined.
- Nuclear matrix elements must be known from calculations
- Observation of $0\nu\beta\beta$-decay of several nuclei a possibility to test NME calculations