Neutrino oscillations

a historical overview and its projection

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Topics

1 **Base fermions and scalars in SO10**

   neutrinos are unlike charged fermions - Ettore Majorana

2 **Neutrino 'mass from mixing' in vacuo and matter**

   neutrinos oscillate like neutral Kaons (yes, but how ?) - Bruno Pontecorvo

3 **Some perspectives**

Charged fermions are not like neutrinos [1]

We shall consider - 'pour fixer les idees' - 3 fermion families in the (left-) chiral basis,

forming a substrate for the local gauge group

SL (2,C) [or SO (1,3)] x SO10

$$\begin{pmatrix} u^1 & u^2 & u^3 & & \nu & \vert & N & & \hat{u}^1 & \hat{u}^2 & \hat{u}^3 \\ d^1 & d^2 & d^3 & & l^- & \vert & l^+ & & \hat{d}^1 & \hat{d}^2 & \hat{d}^3 \end{pmatrix}_F^{\mathring{\gamma}}$$

$f_F^{\mathring{\gamma}}$ $\mathring{\gamma} = 1, 2$

$F = e, \mu, \tau$ —> Fig. 1

[1] Ettore Majorana, ' Teoria simmetrica dell elettrone e positrone ' , Nuovo Cimento 14 (1937) 171.

I shall cite two sentences from ref. [1] :

”Per quanto riguarda gli elettroni e i positroni, da essa (via) si può veramente attendere soltanto un progresso formale ... Vedremo infatti che è perfettamente possibile costruire, nella maniera più naturale,

una teoria delle particelle neutre elementari

senza stati negativi.”

(”As far as electrons and positrons are concerned from this (path) one may expect only a formal progress ... We will see in fact that it is perfectly possible to construct, in the most general manner,

a theory of neutral elementary particles

without negative states.” [a])

[a] ... upon normal ordering .

But the real content of the paper by E. M. (1937) is in the formulae, exhibiting the 'oscillator decomposition' of spin 1/2 fermions

as seen and counted by gravity, 1 by 1 and doubled through the 'external' SO2 symmetry associated with electric charge [a]

The left chiral notation shall be

$$(1) \quad \begin{array}{rl} (f_{k})^{\dot{\gamma}}_{F} \; ; \; \dot{\gamma} = 1, 2 & : \text{ spin projection} \\ F = I, II, III & : \text{ family label} \\ k = 1, \cdots, 16 & : \text{ SO10 label} \end{array}$$

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[a] [2] P. A. M. Dirac, Proceedings of the Cambridge Philosophical Society, 30 (1924) 150. Paul Dirac shall be excused for starting the count at 2 for 'elettrone e positrone' .

Lets call the above extension of the standard model the 'minimal nu-extended SM' . [a]

$$\begin{pmatrix} \bullet & \bullet & \bullet & \nu & | & \mathcal{N} & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \ell & | & \widehat{\ell} & \bullet & \bullet & \bullet \end{pmatrix}^{\dot{\gamma}}_{F=e\,,\,\mu\,,\,\tau}$$

(2)

$$\downarrow$$

$$\begin{pmatrix} \nu & \mathcal{N} \\ \ell & \widehat{\ell} \end{pmatrix}^{\dot{\gamma}}_{F=e\,,\,\mu\,,\,\tau}$$

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a [3] Harald Fritzsch and Peter Minkowski, "Unified interactions of leptons and hadrons", Annals Phys.93 (1975) 193 and Howard Georgi, "The state of the art - gauge theories", AIP Conf.Proc.23 (1975) 575.

The right-chiral base fields are then associated to **1 for 1**

$$\left(f^{\,*}_{\,k} \right)_{\,F\,\alpha} \;=\; \varepsilon_{\,\alpha\gamma} \left[\left(f_{\,k} \right)^{\,\dot{\gamma}}_{\,F} \right]^{\,*}$$

(3)

$$\left(\varepsilon \;=\; i\,\sigma_{\,2} \right)_{\,\alpha\gamma} \;=\; \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

The matrix ε is the symplectic ($Sp\,(1)$) unit, as implicit in Ettore Majorana's original paper [1] .

The local gauge theory is based on the gauge (sub-) group

(4) $$SL\,(\,2\,,\,C\,)\,\times\,SU3_{\,c}\,\times\,SU2_{\,L}\,\times\,U1_{\,\mathcal{Y}}$$

... why ? why 'tilt to the left' ? we sidestep a historical overview here !

1 a) Yukawa interactions and mass terms

The doublet(s) of scalars are related to the 'tilt to the left' .

$$(5) \qquad \begin{pmatrix} \nu & \mathcal{N} \\ \ell & \widehat{\ell} \end{pmatrix}_F \leftrightarrow \begin{pmatrix} \varphi^{\,0} & \Phi^{\,+} \\ \varphi^{\,-} & \Phi^{\,0} \end{pmatrix} = z$$

The green entries in eq. (4) denote singlets under $SU2_{\,L}$.

The quantity z is associated with the quaternionic or octonionic structure inherent to the $(\,2\,,\,2\,)$ representation of $SU2_{\,L} \otimes SU2_{\,R}$ (beyond the electroweak gauge group) $[\,4\,]$ [a] .

[a] e.g. $[\,4\,]$ F. Gürsey and C.H. Tze , "On the role of division- , jordan- and related algebras in particle physics", Singapore, World Scientific (1996) 461.

The Yukawa couplings are of the form (notwithstanding the quaternionic or octonionic structure of scalar doublets)

$$\mathcal{H}_Y = \left[(\varphi^0)^*, (\varphi^-)^* \right] \lambda_{F' F} \times$$

$$\times \left\{ \varepsilon_{\dot\gamma \dot\delta} \mathcal{N}^{\dot\delta}_{F'} \begin{bmatrix} \nu^{\dot\gamma} \\ \ell^{\dot\gamma} \end{bmatrix}_F \right\} + h.c. \tag{6}$$

$$\mathcal{N}_{\dot\gamma F'} = \varepsilon_{\dot\gamma \dot\delta} \mathcal{N}^{\dot\delta}_{F'} \; ; \; \varepsilon_{\dot\gamma \dot\delta} = \overline{\varepsilon_{\gamma \delta}} = \varepsilon_{\gamma \delta}$$

The only allowed Yukawa couplings by $SU2_L \otimes U1_{\mathcal{Y}}$ invariance are those in eq. (6) , with arbitrary complex couplings $\lambda_{F' F}$.

Spontaneous breaking of $SU2_L \otimes U1_{\mathcal{Y}}$ through the vacuum expected value(s)

$$\langle \Omega | \begin{pmatrix} \varphi^0 & \Phi^+ \\ \varphi^- & \Phi^0 \end{pmatrix} (x) | \Omega \rangle =$$

(7)

$$= \langle z(x) \rangle = \begin{pmatrix} v_{ch} \, (v^u_{ch}) & 0 \\ 0 & v_{ch} \, (v^d_{ch}) \end{pmatrix}$$

$$v_{ch} = \frac{1}{\sqrt{2}} \left(\sqrt{2} \, G_F \right)^{-1/2} = 174.1 \text{ GeV}$$

independent of the space-time point x [a],

[a] The implied parallelizable nature of $\langle z(x) \rangle$ is by far not trivial and relates in a wider context including triplet scalar representations to potential (nonabelian) monopoles and dyons. (no h.o.)

induces a neutrino mass term through the Yukawa couplings $\lambda_{F' F}$ in eq. (6)

$$ {}_{F'}\mathcal{N} \, \nu_F = \mathcal{N}_{\dot\gamma F'} \, \nu^{\dot\gamma}_{F} = \nu_{\dot\gamma F} \, \mathcal{N}^{\dot\gamma}_{F'} $$

(8)
$$ \mu_{F' F} = v_{ch} \, \lambda_{F' F} $$

$$ \rightarrow \mathcal{H}_{\mu} = {}_{F'}\mathcal{N} \, \mu_{F' F} \, \nu_F + h.c. = \nu^T \mu^T \mathcal{N} + h.c. $$

The matrix μ defined in eq. (8) is an arbitrary complex 3×3 matrix, analogous to the similarly induced mass matrices of charged leptons and quarks. In the setting of primary SO10 breakdown, a general (not symmetric) Yukawa coupling $\lambda_{F' F}$ implies the existence in the scalar sector of at least two irreucible representations $(16) \oplus (120)$ [a].

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[a] key question $\rightarrow$ a 'drift' towards unnatural complexity ? It becomes even worse including the heavy neutrino mass terms : 256 (complex) scalars.

2 a) 'Mass from mixing' in vacuo [5] - [7] ($\rightarrow$)
or 'Seesaw' [8] - [11] ($\rightarrow$)

neutrinos oscillate like neutral Kaons (yes, but how ?) - Bruno Pontecorvo [a]

The special feature, pertinent to (electrically neutral) neutrinos is, that the $\nu -$ extending degrees of freedom $\mathcal{N}$ are singlets under the whole SM gauge group $G_{SM} = SU3_c \otimes SU2_L \otimes U1_{\mathcal{Y}}$, in fact remain singlets under the larger gauge group $SU5 \supset G_{SM}$.
This allows an arbitrary (Majorana-) mass term, involving the bilinears formed from two $\mathcal{N}$-s.
In the present setup (minimal ν-extended SM) the full neutrino mass term is thus of the form $\rightarrow$

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[a] We will come back to the clearly original idea in 1957 of Bruno Pontecorvo [12 −] but let me first complete the 'flow of thought' embedding neutrino masses in SO10.

(9)

$$\mathcal{H}_{\mathcal{M}} = \frac{1}{2}\,[\,\nu\,\mathcal{N}\,]\;\mathcal{M}\begin{bmatrix}\nu\\ \mathcal{N}\end{bmatrix} + h.c.$$

$$\mathcal{M} = \begin{pmatrix} 0 & \mu^{T} \\ \mu & M \end{pmatrix} \;;\; \mathcal{M} = \mathcal{M}^{T} \rightarrow M = M^{T}$$

Again within primary SO10 breakdown the full $\mathcal{M}$ extends the scalar sector to the representations $(16) \oplus (120) \oplus (126)$ [a].

[a] It is from here where the discussion – to the best of my knowledge – of the origin and magnitude of the light neutrino masses (re-) started in 1974 as documented on the next slide.

[5] Harald Fritzsch, Murray Gell-Mann and Peter Minkowski, ”Vector - like weak currents and new elementary fermions” , Phys.Lett.B59 (1975) 256.

[6] Harald Fritzsch and Peter Minkowski, ”Vector - like weak currents, massive neutrinos, and neutrino beam oscillations” , Phys.Lett.B62 (1976) 72. ([5] and [6] in the the general vectorlike situation.)

and for ’our world, tilted to the left’

[7] Peter Minkowski, ”$\mu \rightarrow e\gamma$ at a rate of one out of 1-billion muon decays ?” , Phys.Lett.B67 (1977) 421.

Correct derivations were subsequently documented in [8] - [11]

[8] Murray Gell-Mann, Pierre Ramond and Richard Slansky, ”Complex spinors and unified theories” , published in Supergravity, P. van Nieuwenhuizen and D.Z. Freedman (eds.), North Holland Publ. Co., 1979 and in Stony Brook Wkshp.1979:0315 (QC178:S8:1979).

[9] Tsutomu Yanagida, ”Horizontal symmetry and masses of neutrinos” , published in the Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories, O. Sawada and A. Sugamoto (eds.), Tsukuba, Japan, 13-14 Feb. 1979, and in (QCD161:W69:1979) .

[10] Shelley Glashow, ”Quarks and leptons” , published in Proceedings of the Cargèse Lectures, M. Lévy (ed.), Plenum Press, New York, 1980.

[11] Rabindra Mohapatra and Goran Senjanovič, ”Neutrino mass and spontaneous parity violation” , Phys.Rev.Lett.44 (1980) 912.

We resume the discussion of the mass term in eq. (9).
Especially the 0 entry needs explanation. It is an exclusive property of the minimal ν-extension assumed here.
Since the 'active' flavors ν_F all carry $I_{3\,w} = \frac{1}{2}$ terms of the form

$$(10) \qquad \frac{1}{2}\ _{F'}\nu\,\chi_{F'F}\,\nu_F = \frac{1}{2}\,\nu^T \chi\,\nu \ ; \ \chi = \chi^T$$

cannot arise as Lagrangean masses, except induced by an I_w-triplet of scalars, developing a vacuum expected value independent from the doublet(s) [a].

from "The apprentice magician" by Goethe : 'The shadows I invoked , I am unable to get rid of now !'

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[a] key questions $\rightarrow$ quo vadis ? is this a valid explanation of the 'tilt to the left' ? no , at least insufficcient !

2 0) Neutrino oscillations - historical overview

The idea that light neutrinos have mass and oscillate goes back to Bruno Pontecorvo, but starting with (para-) muonium - antimuonium oscillations $[\,12\,]$ [a] - like $K^{\,0} \leftrightarrow \overline{K}_{\,0}\,[\,13\,]$ [b]. Assuming CP conservation there are two equal mixtures of $\mu^{\,-}\,e^{+}$ and $\mu^{\,+}\,e^{-}$ with opposite CP values $\pm$ (at rest and using a semiclassical description of quantum states)

$$|\,(\,e\mu\,)_{\,\pm}\,;\,\tau = 0\,\rangle \;=\; \frac{1}{\sqrt{2}}\,\left(\,|\,(\,e^{\,-}\mu^{\,+}\,)\,\rangle \,\mp\, |\,(\,e^{\,+}\mu^{\,-}\,)\,\rangle\,\right)_{\,\tau=0}$$

$$|\,(\,e^{\,+}\mu^{\,-}\,)\,\rangle \;=\; \widehat{C}\;|\,(\,e^{\,-}\mu^{\,+}\,)\,\rangle$$

(11)

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[a] $[\,12\,]$ Bruno Pontecorvo, "Mesonium and antimesonium", JETP (USSR) 33 (1957) 549, english translation Soviet Physics, JETP 6 (1958) 429.

[b] $[\,13\,]$ Murray Gell-Mann and Abraham Pais, Phys. Rev. 96 (1955) 1387, introducing τ .

For the leptonium case the rest system is a good appoximation. The evolution of the CP $\pm$ states is then characterized by

$$\widehat{m}_{\alpha} = m_{\alpha} - \frac{i}{2}\Gamma_{\alpha} \; ; \; \alpha = \pm$$ **with**

$$|\,(e^{-}\mu^{+})\,\rangle = |\,1\,\rangle \rightarrow_{\tau}$$

$$\frac{1}{\sqrt{2}}\left(|+;\tau=0\rangle\, e^{-i\,\widehat{m}_{+}\tau} + |-;\tau=0\rangle\, e^{-i\,\widehat{m}_{-}\tau}\right)$$

$$|\,(e^{+}\mu^{-})\,\rangle = |\,2\,\rangle \rightarrow_{\tau}$$

$$\frac{1}{\sqrt{2}}\left(-\,|+;\tau=0\rangle\, e^{-i\,\widehat{m}_{+}\tau} + |-;\tau=0\rangle\, e^{-i\,\widehat{m}_{-}\tau}\right)$$

(12)

This reconstructs to $\rightarrow$

$$|1\rangle \to_\tau \quad E_+(\tau)\,|1\rangle - E_-(\tau)\,|2\rangle$$

(13) $$|2\rangle \to_\tau \quad -E_-(\tau)\,|1\rangle + E_+(\tau)\,|2\rangle$$

$$E_\pm(\tau) = \frac{1}{2}\left(e^{-i\,\widehat{m}_+\,\tau} \pm e^{-i\,\widehat{m}_-\,\tau}\right)$$

and leads to the transition relative probabilities indeed identical to the $K^0 \to |1\rangle\,;\overline{K}_0 \to |2\rangle$ system.

$$d\,p_{1\leftarrow 1} = d\,p_{2\leftarrow 2} = |E_+(\tau)|^2\,d\tau$$

$$d\,p_{2\leftarrow 1} = d\,p_{1\leftarrow 2} = |E_-(\tau)|^2\,d\tau$$

$$|E_\pm(\tau)|^2 =$$

$$= \frac{1}{2}\,e^{-\frac{1}{2}(\Gamma_+ + \Gamma_-)\tau}\left(\cosh\frac{1}{2}\Delta\Gamma\tau \pm \cos\Delta m\,\tau\right) \quad \to$$

(14)

The term $\cos \Delta m \tau$ in eq. (14) indeed signals $(e\widehat{\mu}) \leftrightarrow (\widehat{e}\mu)$ oscillations, with

$$\Delta m = m_+ - m_- \; ; \; \Delta \Gamma = \Gamma_+ - \Gamma_-$$

$$\Delta m = O \left[\left(\frac{\alpha \, m_e \, m_{\nu_e \nu_\mu}}{v^2} \right)^2 m_\mu \right] \sim 4 \, . \, 10^{-41} \text{ MeV}$$

$$\tau_{osc} = (2\pi) / \Delta m \sim 10^{20} \text{ sec} = 3.3 \, . \, 10^{12} \text{ y}$$

(15)

"Erstens kommt es anders, zweitens als man denkt." [a] "First it happens differently, second as one thinks."

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[a] Not only this is clearly unobservable, but eq. (14) ignores CP violation (no h.o.) , and details of neutrino mass and mixing, which induces Δm in eqs. (14-15) .

What is to be remembered from ref. [14] is the i d e a of neutrino oscillations, expressed in the *corrected* sentence :

"The effects due to *neutrino flavor transformations* may not be observable in the laboratory, owing to the large R, but they will take place on an astronomical scale."

The $(V - A) \times (V - A)$ form of the Fermi interaction [15] [b], which subsequently clarified the structure of neutrino emission and absorption, was documented almost contemporaneously.

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[a] [14] Bruno Pontecorvo, "Inverse β processes and nonconservation of lepton charge", JETP (USSR) 34 (1957) 247, english translation Soviet Physics, JETP 7 (1958) 172.

[b] [15] Richard Feynman and Murray Gell-Mann, "Theory of Fermi interaction", Phys.Rev.109 1. January (1958) 193, (no h.o.) .

$\Delta m \, \tau$ from rest system to beam system [16] [a]

There is time dilatation from rest system to beam system, and also we express time in the beam system by distance ($c = 1$)

(16) $$\tau \rightarrow \frac{d}{\gamma \beta} \; ; \; \gamma^{-1} = \sqrt{1 - v^2} \; ; \; \beta = v$$ [b]

Then we replace Δm , for 12 beam oscillations

(17) $$\Delta m = \frac{\Delta m^2}{2 \langle m \rangle} \; ; \quad \begin{aligned} \Delta m^2 &= m_1^2 - m_2^2 \\ \langle m \rangle &= \tfrac{1}{2} (m_1 + m_2) \end{aligned}$$

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[a] [16] In notes to Jack Steinberger, lectures on "Elementary particle physics", ETHZ, Zurich WS 1966/67, not documented (again zigzag in time) .

[b] key question $\rightarrow$ which v ?

$\rightarrow$

Thus we obtain , for *any* 12 oscillation phenomenon

$$(18) \qquad \Delta m \, \tau = \frac{\Delta m^2}{2 \langle m \rangle \beta \gamma} d \; ; \; \langle m \rangle \, \beta \gamma = \langle p \rangle$$

It is apparently clear that $\langle m \rangle \; \beta \gamma = \langle p \rangle$ represents the average beam momentum, yet this is not really so. Lets postpone the questions (which $\langle p \rangle$? - which d ?) . From eq. (18) it follows

$$(19) \qquad \Delta m \, \tau = \frac{\Delta m^2}{2 \langle p \rangle} d \; \rightarrow \; \cos \left(\frac{\Delta m^2}{2 \langle p \rangle} d \right)$$

The oscillation amplitude in vacuo (eq. 19) is well known, yet it contains 'subtleties' . $\rightarrow$

$\Delta m^2 d / (2 \langle p \rangle)$: what means what ?

The *semiclassical* intuition from beam dynamics and optical interference is obvious. A well collimated and within $\Delta |\vec{p}| / |\langle \vec{p} \rangle|$ [a] 'monochromatic' beam is considered as a classical line, lets say along the positive z-axis, defining the mean direction from a definite production point ($\vec{x} = 0$) towards a detector, at distance d .

But the associated operators for a single beam quantum

$$\widehat{p}_z , \widehat{z} \rightarrow \Delta \widehat{p}_z \Delta \widehat{z} \geq \frac{1}{2} \tag{20}$$

are subject to the uncertainty principle (using units $\hbar = 1$) .

The same is true for energy and time. Yet we are dealing $\rightarrow$

[a] – or any similar definition of beam momentum spread –

in oscillations – with single quantum interference – and thus the spread from one beam quantum to the next is only yielding a 'good guess' of the actual expectation values, e.g. appearing in eq. (20) . The quantity $\langle \, p \, \rangle$ in the expression for the phase

(21) $$\Delta \, m^{\,2} \, d \, / \, (\, 2 \, \langle \, p \, \rangle \,)$$

essentially presupposes the single quantum production wave function, e.g. in 3 momentum space in a given fixed frame, propagating from a production time $t_{\,P}$ to a specific detection space-time point $x_{\,D}$ and characterized accounting for all quantum mechanical uncertainties by the distance d . In this framework $\langle \, p \, \rangle$ stands for the so evaluated single quantum expectation value [a] .

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[a] This was the content of my notes in ref. [16] (1966) .

h.o. →

This was implicit in refs. (e.g.) [5] – [7], and became obvious in discussing matter effects, specifically for neutrino oscillations (e.g. in the sun) .

The ensuing is an *incomplete* attempt of a historical overview, going zigzag in time, starting with ref. [17] [a].
Just mention is due to two papers : Shalom Eliezer and Arthur Swift [18] [b] and Samoil Bilenky and Bruno Pontecorvo [19] [c],

where the phase argument $\Delta m^2 d / (2 \langle p \rangle)$ appears correctly.

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a [17] Carlo Giunti, "Theory of neutrino oscillations", hep-ph/0409230, in itself a h.o.

b [18] Shalom Eliezer and Arthur Swift, "Experimental consequences of $\nu_e - \nu_\mu$ mixing in neutrino beams", Nucl. Phys. B105 (1976) 45, submitted 28. July 1975.

c [19] Samoil Bilenky and Bruno Pontecorvo, "The lepton-quark analogy and muonic charge", Yad. Fiz. 24 (1976) 603, submitted 1. January 1976.

Also in 1976 a contribution by Shmuel Nussinov [20] appeared [a].

Matter effects - MSW for neutrinos [21] [b], [22] [c]

The general remark hereto is

"Every conceivable coherent or incoherent phenomenon involving photons, is bound to happen (and more) with neutrinos."

$\rightarrow$ refraction, double refraction, Čerenkov radiation, $\cdots$ [23] [d].

The forward scattering amplitude and refractive index relation is (a semiclassical one) $\rightarrow$

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a [20] Shmuel Nussinov, "Solar neutrinos and neutrino mixing", Phys.Lett.B63 (1976) 201, submitted 10. May 1976.

b [21] Lincoln Wolfenstein, "Neutrino oscillations in matter", Phys.Rev.D17 (1978) 2369.

c [22] Stanislav Mikheyev and Alexei Smirnov, Sov. J. Nucl. Phys. 42 (1985) 913.

d [23] see e.g. Arnold Sommerfeld, "Optik", "Elektrodynamik", "Atombau und Spektrallinien", Akademische Verlagsgesellschaft, Geest und Ko., Leipzig 1959.

plane wave distortion in the z-direction

$$f_0 \equiv f_{forward}^{lab} = (8\pi m_{target})^{-1} T_{forward}$$

$$Im f_0 = (k_{lab} / (4\pi)) \sigma_{tot} ; \quad k_{lab} \rightarrow k$$

$$e^{ikz} \rightarrow e^{inkz} = e^{ikz} e^{i(2\pi/k)\varrho_N f_0 z}$$

$$= e^{ikz} e^{i[(2\pi/k^2)\varrho_N f_0] kz}$$

$$n = 1 + (2\pi/k^2)\varrho_N f_0 ; \quad \langle v \rangle_{mat.} \sim 1/(Re\, n) \lesseqgtr ? \; 1$$

ϱ_N = mean number density of (target-) matter [a]

T = invariantly normalized (elastic-) scattering amplitude

(22)

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[a] key question → which is the fully quantum mechanical description ?

for neutrinos [24] [a] at low energy :

$$\mathcal{H}_{\nu} \sim 2\sqrt{2}\, G_F \left(\begin{array}{c} \overline{\nu}_{\alpha} \gamma_L^{\mu} \nu_{\beta} \, \overline{\ell}_{\beta} \gamma_{\mu L} \ell_{\alpha} \\ + \overline{\nu}_{\alpha} \gamma_L^{\mu} \nu_{\alpha} \, j_{\mu n}(\ell, q)\, \varrho \\ + \frac{1}{4} \overline{\nu}_{\alpha} \gamma_L^{\mu} \nu_{\alpha} \, \overline{\nu}_{\beta} \gamma_L^{\mu} \nu_{\beta} \, \varrho \end{array} \right) \tag{23}$$

$\alpha, \beta = I, II, III$ for family ; ϱ : e.w. neutral current parameter

The second and third [b] terms on the r.h.s. of eq. (23) – in matter consisting of hadrons and electrons – do not distinguish

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[a] [24] Hans Bethe, "A possible explanation of the solar neutrino puzzle", Phys.Rev.Lett.56 (1986) 1305, indeed, tribute – to many who 'really did it' – and to a pioneer of solar physics and beyond (no h.o.) .

[b] the latter induces – tiny – matter distortions on relic neutrinos , maybe it is worth while to work them out ?

between neutrino flavors. So the relative distortion of ν_e – by electrons at rest – is

$$\Delta_e \mathcal{H}_\nu \to \sqrt{2}\, G_F\, \nu_e^{\dot\beta\,*}\, \nu_e^{\dot\beta}\, \langle e^* e \rangle_e$$

$$= \left(\sqrt{2}\, G_F\, \varrho_{n_e} \right) \nu_e^{\dot\beta\,*}\, \nu_e^{\dot\beta} \tag{24}$$

$$K_e = \sqrt{2}\, G_F\, \varrho_{n_e}\, ; \left(K_e \to -K_e \text{ for } e^- \to e^+ \right)$$

The spinor field equation in the above semiclassical approximation in (chiral) basis becomes – suppressing all indices – and allowing for an $\vec{x}$ dependent electron density

$$\left.\begin{array}{l}(i\partial_t - \kappa)\,\nu = \mathcal{M}^{\dagger}\,\widetilde{\nu}\\ (i\partial_t + \kappa)\,\widetilde{\nu} = \mathcal{M}\,\nu\end{array}\right. \;;\; i\partial_t \to E$$

$$\kappa = K_e P_e - \tfrac{1}{i}\vec{\sigma}\,\vec{\nabla}\,\P \;;\; \nu = \nu_f^{\dot\beta} \;;\; \widetilde{\nu} = \varepsilon_{\alpha\beta}\left(\nu_f^{\dot\beta}\right)^{*} \tag{25}$$

$$f = 1,\cdots,6 \;;\; P_e = \delta_{ef}\,\delta_{ef'} \;;\; \P = \delta_{ff'} \;;\; \mathcal{M} = \mathcal{M}_{ff'}$$

$(\nu, \widetilde{\nu})\,(t, \vec{x})$: **fields** ; $*$: **hermitian conjugation**

$$K_e = K_e(\vec{x}) = \sqrt{2}\,G_F\,\varrho_{n_e}(\vec{x})$$

Here we substitute fields by wave functions $\rightarrow$

$$(\nu , \widetilde{\nu}) \to e^{-iEt} (\nu , \widetilde{\nu}) (E , \vec{x}) \quad \Big| ^{a}$$

$$\to (E - \kappa)\, \nu = \mathcal{M}^{\dagger} \widetilde{\nu} \; ; \; (E + \kappa)\, \widetilde{\nu} = \mathcal{M} \nu \tag{26}$$

From eq. (26) we obtain the 'squared' form

$$(E^2 - \kappa^2 - \mathcal{M}^{\dagger} \mathcal{M})\, \nu = [\kappa , \mathcal{M}^{\dagger}]\, \widetilde{\nu}$$

$$(E^2 - \kappa^2 - \mathcal{M} \mathcal{M}^{\dagger})\, \widetilde{\nu} = [\mathcal{M} , \kappa]\, \nu$$

$$[\mathcal{M} , \kappa] = K_e [\mathcal{M} , P_e] \; ; \; [\kappa , \mathcal{M}^{\dagger}] = [\mathcal{M} , \kappa]^{\dagger} \quad \Big| ^{b}$$

$$\kappa^2 = -\Delta - 2 P_e K_e \tfrac{1}{i} \vec{\sigma} \vec{\nabla} - P_e \vec{\sigma} (\tfrac{1}{i} \vec{\nabla} K_e) + P_e K_e^2 \tag{27}$$

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[a] The quantities $\widetilde{\nu}(E , \vec{x})$ are not related to complex conjugate entries for $\nu(E , \vec{x})$.

[b] The purple quantities in eq. (27) give (e.g. in the sun) negligible effects for the light flavors $\to 0$.

In eqs. (26-27) the mainly neutrinos have negative helicity, which can be specified precisely if the purple quantities are ignored, whereas the mainly antineutrinos carry positive helicity, in the ultrarelativistic limit $p = |\vec{p}| \gg |m|$.

$$(28) \quad \kappa^2 \to p^2 \pm 2\,p\,K_e\,P_e \; : \; \begin{cases} + \text{ for neutrinos} \\ - \text{ for antineutrinos} \end{cases}$$

The mass diagonalization yields correspondingly for the mixing in vacuo approximatively

$$(29) \quad \mathcal{M}^\dagger \mathcal{M} \to \overline{u}\, m^2_{diag}\, \overline{u}^{-1} \; ; \; \mathcal{M}\mathcal{M}^\dagger \to u\, m^2_{diag}\, u^{-1}$$

In eq. (29) the red quantities refer to the three light flavors [a].

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[a] So for real (i.e. orthogonal) u , neutrinos distorted by electrons react identically to antineutrinos relative to positrons .

[a]

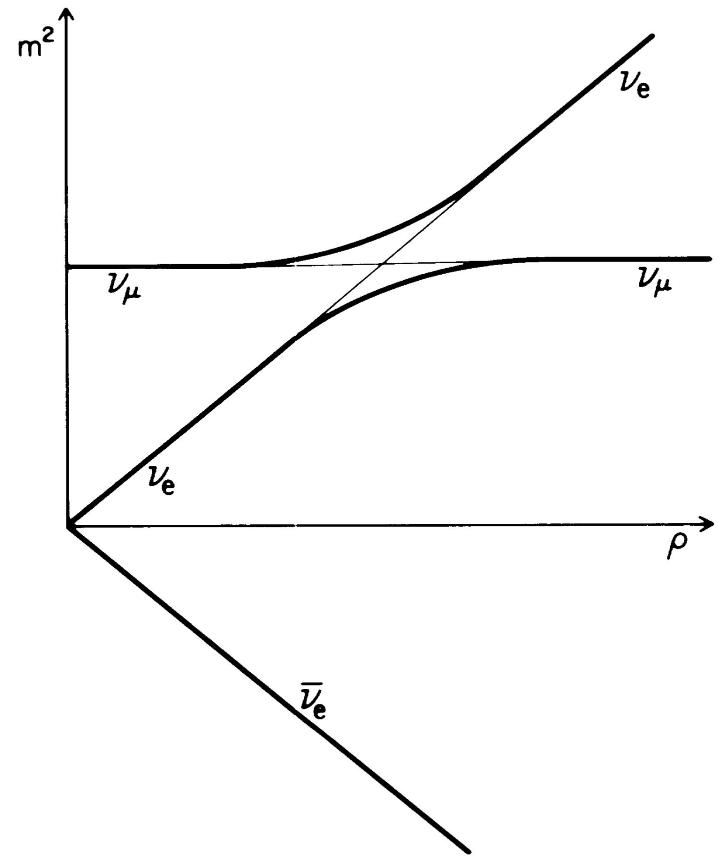

FIG. 1. The masses of two flavors of neutrinos as a function of density. The curves nearly cross at one point. The electron-antineutrino mass $\overline{\nu}_e$ is also shown.

[a] **From Hans Bethe, ref. [24], with apologies to Mikheyev and Smirnov and many.**

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(Further) references to the $\odot$ LMA solution [a]

[25] Alexei Smirnov, "The MSW effect and matter effects in neutrino oscillations", hep-ph/0412391.

$$\Delta m^2_\odot = 7.9 \, . \, 10^{-5} \, \text{ev}^2 \;\; ; \;\; \tan^2 \vartheta_\odot = 0.40 \;\; ; \;\; \vartheta \sim 32.3^\circ \tag{30}$$

[26] Serguey Petcov, "Towards complete neutrino mixing matrix and CP-violation", hep-ph/0412410.

$$\Delta m^2_\odot = \left(7.9^{+0.5}_{-0.6} \right) . \, 10^{-5} \, \text{ev}^2 \;\; ; \;\; \tan^2 \vartheta_\odot = 0.40^{+0.09}_{-0.07} \tag{31}$$

Fig. $\rightarrow$

[27] John Bahcall and Carlos Pena-Garay, "Global analyses as a road map to solar neutrino fluxes and oscillation parameters", JHEP 0311 (2003) 004, hep-ph/0305159.

[28] Gian Luigi Fogli, Eligio Lisi, Antonio Marrone, Daniele Montanino, Antonio Palazzo and A.M. Rotunno, "Neutrino oscillations: a global analysis", hep-ph/0310012.

[28] P. Aliani, Vito Antonelli, Marco Picariello and Emilio Torrente- Lujan, "The neutrino mass matrix after Kamland and SNO salt enhanced results", hep-ph/0309156.

[29] Samoil Bilenky, Silvia Pascoli and Serguey Petcov, "Majorana neutrinos, neutrino mass spectrum, CP violation and neutrinoless double beta decay. 1. The three neutrino mixing case", Phys.Rev.D64 (2001) 053010, hep-ph/0102265.

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[a] See also many refs. cited therein, (no h.o.) .

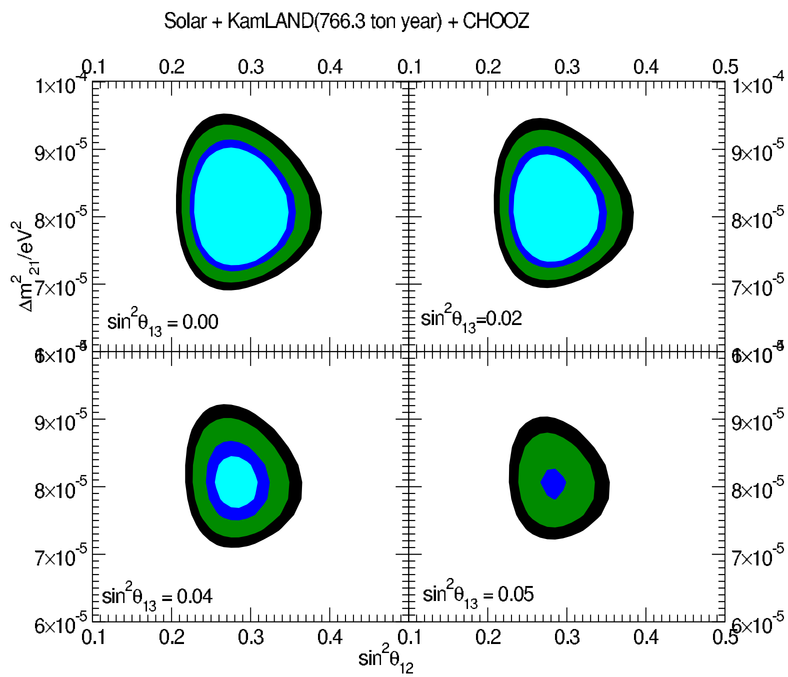

Figure 1. The 90%, 95%, 99% and 99.73% C.L. allowed regions in the Δm^2_{21} -$\sin^2\theta_{12}$ plane, obtained in a 3-ν oscillation analysis of the solar neutrino, KL and CHOOZ data [15].

Experimental references to the ⊙ LMA solution [a]

[30] Bruce Cleveland, Timothy Daily, Raymond Davis, James Distel, Kenneth Lande, Choon-kyu Lee, Paul Wildenhain and Jack Ullman, "Measurement of the solar electron neutrino flux with the Home-stake chlorine detector", Astrophys. J. 496 (1998) 505.

reaction : $\nu_{e\odot} + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$

[31] V. Gavrin, "Results from the Russian American gallium experiment", for the SAGE collaboration, J.N. Abdurashitov et al., J. Exp. Theor. Phys. 95 (2002) 181, astro-ph/0204245.

J.N. Abdurashitov, V.N. Gavrin, S.V. Girin, V.V. Gorbachev, P.P. Gurkina, T.V. Ibragimova, A.V. Kalikhov, N.G. Khairnasov, T.V. Knodel, I.N. Mirnov, A.A. Shikhin, E.P. Veretenkin, V.M. Vermul, V.E. Yants, G.T. Zatsepin, Moscow, INR

T.J. Bowles, W.A. Teasdale Los Alamos

J.S. Nico, NIST, Wash., D.C.

B.T. Cleveland, S.R. Elliott, J.F. Wilkerson, Washington U., Seattle.

reaction : $\nu_{e\odot} + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$ →

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[a] See also many refs. cited therein, (no h.o.) .

[31] W. Hampel et al., GALLEX collaboration, ”GALLEX solar neutrino observations: results for GALLEX IV”, Phys. Lett. B 447 (1999) 127.

W. Hampel, J. Handt, G. Heusser, J. Kiko, T. Kirsten, M. Laubenstein, E. Pernicka, W. Rau, M. Wojcik, Y. Zakharov, Heidelberg, Max Planck Inst.

R. von Ammon, K.H. Ebert, T. Fritsch, D. Heidt, E. Henrich, L. Stieglitz, F. Weirich, Karlsruhe U., EKP

M. Balata, M. Sann, F.X. Hartmann, Gran Sasso

E. Bellotti, C. Cattadori, O. Cremonesi, N. Ferrari, E. Fiorini, L. Zanotti, Milan U. and INFN, Milan

M. Altmann, F. von Feilitzsch, R. Mössbauer, S. Wanninger, Munich, Tech. U.

G. Berthomieu, E. Schatzman, Cote d'Azur Observ., Nice

I. Carmi, I. Dostrovsky, Weizmann Inst.

C. Bacci, P. Belli, R. Bernabei, S. d'Angelo, L. Paoluzi, Rome U.,Tor Vergata and INFN, Rome

M. Cribier, J. Rich, M. Spiro, C. Tao, D. Vignaud, DAPNIA, Saclay

J. Boger, R.L. Hahn, J.K. Rowley, R.W. Stoenner, J. Weneser, Brookhaven.

reaction : $\nu_{e\odot} + {}^{71}Ga \to {}^{71}Ge + e^-$ like SAGE . $\to$

[32] Y. Fukuda et al., Kamiokande collaboration ”Solar neutrino data covering solar cycle 22”, Phys. Rev. Lett. 77 (1996) 1683.

Y. Fukuda, T. Hayakawa, K. Inoue, K. Ishihara, H. Ishino, S. Joukou, T. Kajita, S. Kasuga, Y. Koshio, T. Kumita, K. Matsumoto, M. Nakahata, K. Nakamura, K. Okumura, A. Sakai, M. Shiozawa, J. Suzuki, Y. Suzuki, T. Tomoeda, Y. Totsuka, Tokyo U., ICRR

K.S. Hirata, K. Kihara, Y. Oyama, KEK, Tsukuba

Masatoshi Koshiba, Tokai U., Shibuya

K. Nishijima, T. Horiuchi, Tokai U., Hiratsuka

K. Fujita, S. Hatakeyama, M. Koga, T. Maruyama, A. Suzuki, Tohoku U.

M. Mori, Miyagi U. of Education

T. Kajimura, T. Suda, A.T. Suzuki, Kobe U.

T. Ishizuka, K. Miyano, H. Okazawa, Niigata U.

T. Hara, Y. Nagashima, M. Takita, T. Yamaguchi, Osaka U.

Y. Hayato, K. Kaneyuki, T. Suzuki, Y. Takeuchi, T. Tanimori, Tokyo Inst. Tech.

S. Tasaka, Gifu U.

E. Ichihara, S. Miyamoto, K. Nishikawa, INS, Tokyo.

reaction : $\nu_{e\odot} + e^- \rightarrow \nu_e + e^-$ $\rightarrow$

[33] Y.Ashie et al., Super-Kamiokande collaboration, "Evidence for an oscillatory signature in atmospheric neutrino oscillation", Phys.Rev.Lett.93 (2004) 101801, hep-ex/0404034 and Y. Suzuki, "Super-Kamiokande: present and future", Nucl.Phys.Proc.Suppl.137 (2004) 5.

M. Goldhaber, Masatoshi Koshiba, J.G. Learned, S. Matsuno, R. Nambu, L.R. Sulak, Y. Suzuki, R. Svoboda, Y. Totsuka, R.J. Wilkes $\cdots$ [a]

reactions : $\nu_{e\odot} + e^- \rightarrow \nu_e + e^-$

$$\begin{matrix}\nu_{e,\mu} \\ \overline{\nu}_{e,\mu}\end{matrix} \quad + H_2O \rightarrow e^{\mp}, \mu^{\mp} + X$$

^8B solar neutrino flux :

$5.05 \left(1 {}^{+0.20}_{-0.16}\right) . 10^{-6} / cm^2 / s$ for BP2000

$5.82 \, (1 \pm 0.23) \, . 10^{-6} / cm^2 / s$ for BP2004

$\rightarrow$

---

a "Wer zählt die Seelen, nennt die Namen, die gastlich hier zusammenkamen."
Friedrich Schiller

"Who counts the souls, relates the names, who met in piece here for the games."

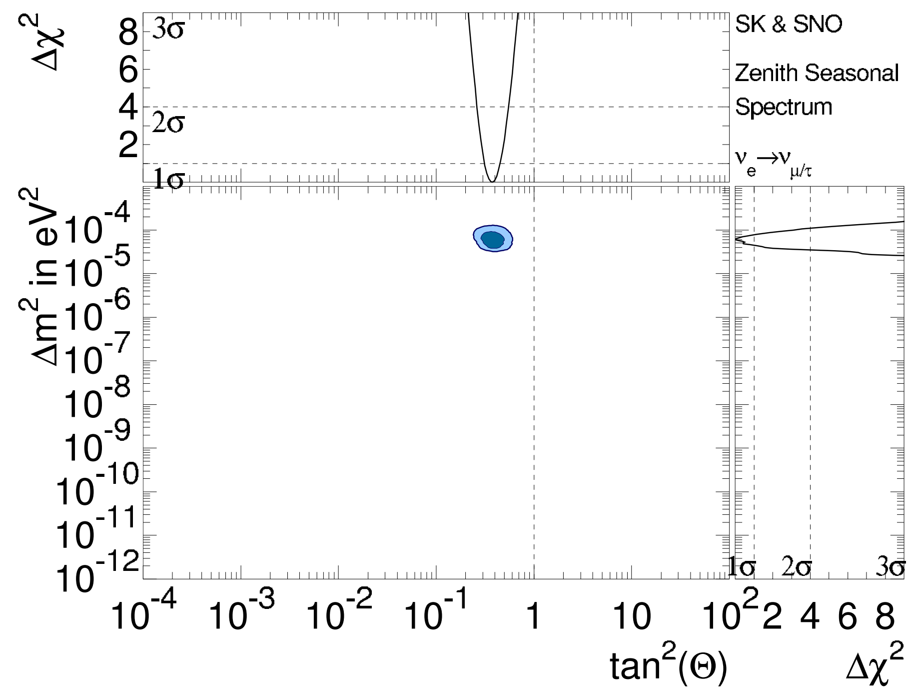


Figure 11. Allowed parameter region obtained by using only Super-K and SNO. The flux constraint from the solar model calculation is not used.

[34] **D. Sinclair for the SNO collaboration, "Recent results from SNO", Nucl.Phys.Proc.Suppl.137 (2004) 150.**

Art McDonald, A. Hamer, J.J. Simpson, D. Sinclair, David Wark $\cdots \leftarrow a$

(32) $$\Delta m^2_\odot = \left(7.1^{+1.0}_{-0.6}\right) . 10^{-5}\,\text{ev}^2 \;;\; \vartheta_\odot = \left(32.5^{+2.4}_{-2.3}\right)^\circ$$

reactions :

$$\nu_{e\odot} + d \rightarrow p + p + e^- \quad \textbf{(CC)}$$

$$\nu_{x\odot} + d \rightarrow p + n + \nu_x \quad \textbf{(NC)}$$

$$\nu_{x\odot} + e^- \rightarrow e^- + \nu_x \quad \textbf{(ES)}$$

Reference(s) to $\overline{\nu}_e \leftrightarrow \overline{\nu}_x$ oscillations

[35] A. Suzuki for the Kamland collaboration, "Results from Kamland", Nucl.Phys.Proc.Suppl.137 (2004) 21.

T. Araki, K. Eguchi, P.W. Gorham, J.G. Learned, S. Matsuno, H. Murayama, Sandip Pakvasa, A. Suzuki, R. Svoboda, P. Vogel $\cdots$ $\leftarrow a$

reaction : $\overline{\nu}_{e\,\odot} + p \to e^- + n$ from $\sim$ 20 reactors $\rightarrow$

leading to the present best estimates (ref. [26]) for 3 flavor oscillations :

$$\Delta m^2_{\odot} = \left(7.9^{+0.5}_{-0.6} \right) . 10^{-5}\,\text{ev}^2 \;\; ; \;\; \tan^2 \vartheta_{\odot} = 0.40^{+0.09}_{-0.07}$$

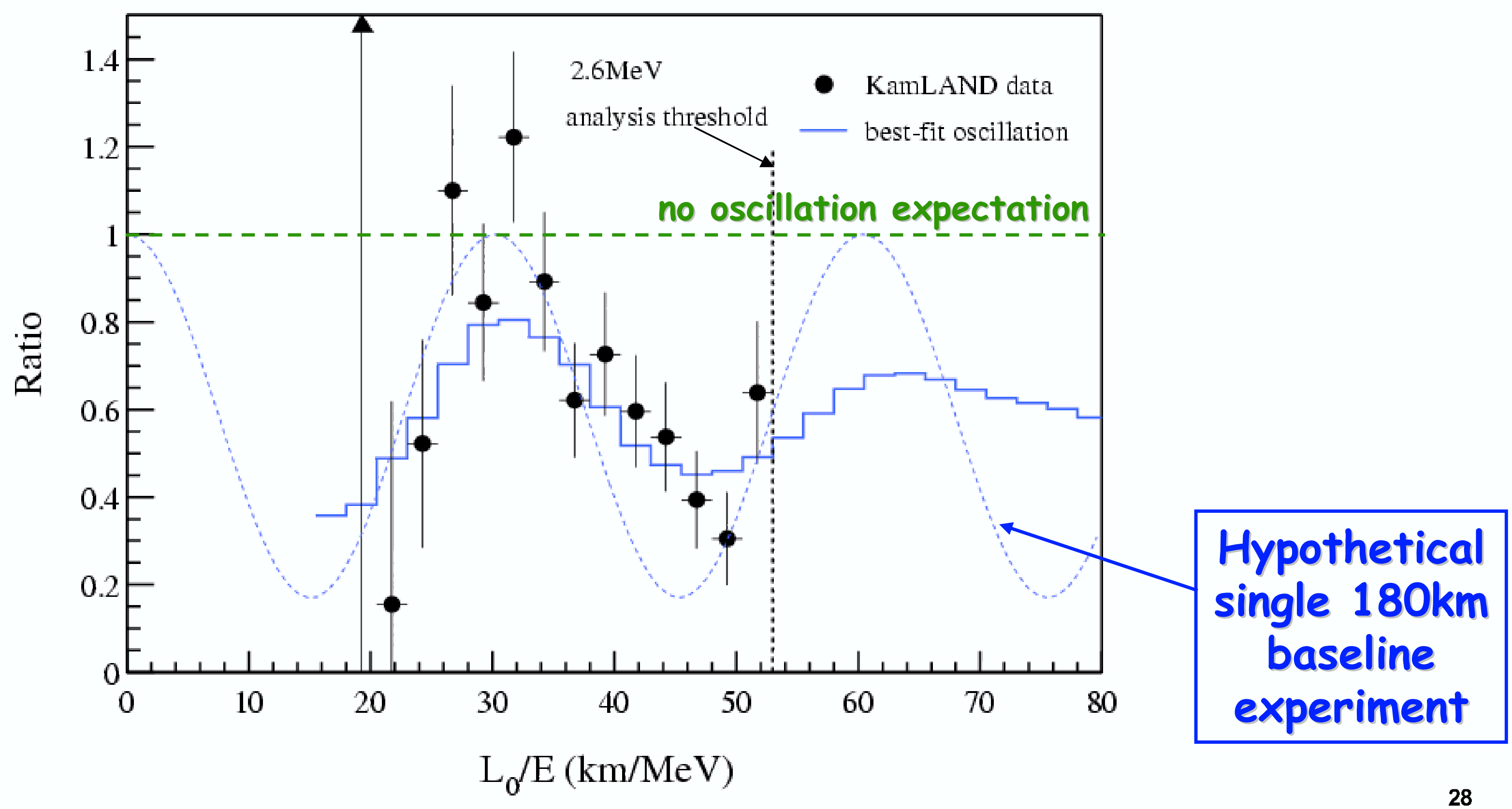
(33) $$| \Delta m^2_{23} | = \left(2.1^{+2.1}_{-0.8} \right) . 10^{-3}\,\text{eV}^2 \;\; ; \;\; \sin^2 2\vartheta_{23} = 1.0_{-0.15}$$

$$\sin^2 \vartheta_{13} \leq 0.05 \quad \text{at} \quad 99.73\,\%\ \text{C.L.}$$

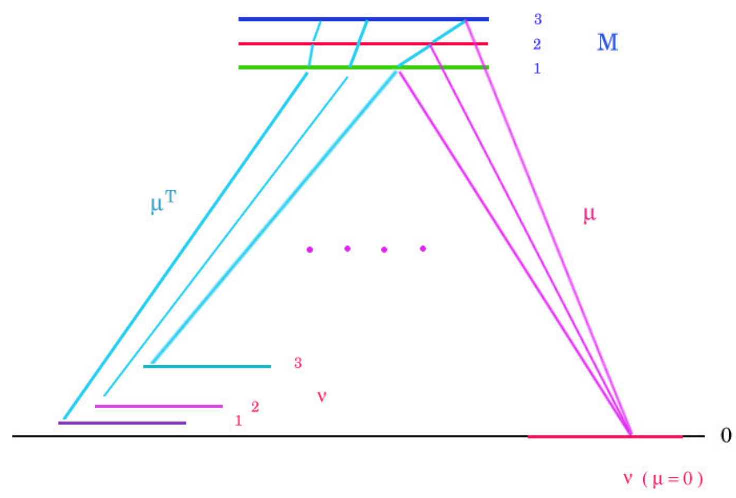
This shall conclude my – necessarily partial – historical overview. What follows must be cut short.

KamLAND uses a range of L and it cannot assign a specific L to each event

Nevertheless the ratio of detected/expected for L_0/E (or $1/E$) is an interesting quantity, as it decouples the ***oscillation pattern*** *from the reactor energy spectrum*



Mass from mixing $\to$ the subtle things



key questions $\to$ which is the scale of M ? $O(10^{10})$ GeV $\to$ is there any evidence for this scale today ? hardly ! $\to$ and what about susy ?

How is the mass matrix of the form in eq. (9) diagonzlized exactly ?

$$\mathcal{M} = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix} = U \, \mathcal{M}_{diag} \, U^T = K \, \mathcal{M}_{bl.diag} \, K^T$$

$$\mathcal{M}_{diag} = \mathcal{M}_{diag}(m_1, m_2, m_3; M_1, M_2, M_3)$$

(34)
$$\mathcal{M}_{bl.diag} = U_0 \, \mathcal{M}_{diag} \, U_0^T \; ; \; U_0 = \begin{pmatrix} u_0 & 0 \\ 0 & v_0 \end{pmatrix}$$

$$U = K \, U_0 = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} \quad \rightarrow$$

$$U = K U_0 \, ; \, K^{-1} \mathcal{M} K^{-1\,T} = \mathcal{M}_{bl.diag.} = \begin{pmatrix} \mathcal{M}_1 & 0 \\ 0 & \mathcal{M}_2 \end{pmatrix}$$

$$U = \begin{pmatrix} (1 + t\,t^\dagger)^{-1/2}\, u_0 & (1 + t\,t^\dagger)^{-1/2}\, t\, v_0 \\ -t^\dagger\, (1 + t\,t^\dagger)^{-1/2}\, u_0 & (1 + t^\dagger t)^{-1/2}\, v_0 \end{pmatrix}$$

$\downarrow \cdots \qquad \rightarrow$

$$\mathcal{M}_1 = -t\, \mathcal{M}_2\, t^T \, ; \quad \left[\begin{array}{lll} \mathcal{M}_1 = u_0\, m_{diag}\, u_0^T & : & \textbf{light 3} \\ \mathcal{M}_2 = v_0\, M_{diag}\, v_0^T & : & \textbf{heavy 3} \end{array} \right]$$

(35)

$$\mathcal{M}_1 = -t\,\mathcal{M}_2\,t^T ; \quad \left[\begin{array}{lll} \mathcal{M}_1 = u_0\, m_{diag}\, u_0^T & : & \text{light } 3 \\ \mathcal{M}_2 = v_0\, M_{diag}\, v_0^T & : & \text{heavy } 3 \end{array} \right]$$

(36)

In eq. (36) all matrices are 3×3, t describes light - heavy mixing generating mass by mixing . [a]

u_0 (unitary) accounts for light-light mixing and v_0 (unitary) for heavy-heavy (re)mixing .

t is 'driven' by μ (in $\mathcal{M}$ $\leftarrow$) and determined $\rightarrow$

---

[a] This is documented in [36] Clemens Heusch and Peter Minkowski, "Lepton flavor violation induced by heavy Majorana neutrinos" , Nucl.Phys.B416 (1994) 3 . Of course 6 by 6 matrices have been diagonalised before .

from the quadratic equation

$$t = \mu^T M^{-1} - t \, \mu \, \bar{t} \, M^{-1} ;$$ to be solved by iteration $\rightarrow$

$$t_{n+1} = \mu^T M^{-1} - t_n \, \mu \, \bar{t}_n \, M^{-1} \; ; \; t_0 = 0$$

$$t_1 = \mu^T M^{-1} , \; , \; t_2 = t_1 - \mu^T M^{-1} \mu \, \mu^\dagger \, \overline{M}^{-1} M^{-1}$$

$$\cdots \; ; \; \lim_{n \rightarrow \infty} t_n = t$$ [a], [b] $\rightarrow$

(37)

---

[a] This is **essentially different** from the mixing of *identical* $SU2_L \times U1$ representations, i.e. charged base fermions.

[b] $[\,37\,]$ Ziro Maki, Masami Nakagawa and Shoichi Sakata, Prog. Theor. Phys. 28 (1962) 870. $\rightarrow$ the PMNS-matrix : The authors assumed (in 1962) only light-light mixing .

Finally lets turn to the mixing matrix u_{11} (eqs. 34-35)

$$u_{11} = (1 + t\,t^{\dagger})^{-1/2}\,u_0 \sim u_0 - \frac{1}{2}\,t\,t^{\dagger}\,u_0$$
$$t\,t^{\dagger} = O\,(\overline{m}\,/\,\overline{M}\,) \sim 10\,\text{meV}\,/\,10^{10}\,\text{GeV} = 10^{-21} \quad (38)$$

The estimate in eq. (38) is very uncertain and assumes among other things $m_1 \sim 1$ meV .

Nevertheless it follows on the same grounds as the smallness of light neutrino masses, that the deviation of u_{11} from u_0 is tiny .

"Much ado about nothing" , William Sakespeare

- - - - - - -

3 Some perspectives

1) Neutrino properties are only to a very small extent open (up to the present) to deductions from oscillation measurements.

2) Notwithstanding this, a significant and admirable experimental effort paired with theoretical analysis has revealed the main two oscillation modes. The matter effect due to Mikheev, Smirnov and Wolfenstein demonstrates another clear form of quantum coherence, over length scales of the solar radius.

3) Key questions remain to be resolved : are all (ungauged or gauged) global charge-like quantum numbers violated ? (B-L) , B , L, individual lepton flavors.

→

4) SO10 served fine (together with susy or without it) to guide ideas, but a genuine unification is as remote as the scales and nature of heavy neutrino flavors, to name only these.

5) I do hope, that not only this workshop ”Neutrino telescopes in Venice” will continue to bring new insights, but also that powerful neutrino telescopes will come into existence, last but not least in the sea.

And the quest for unification remains wide open