What can neutrinos tell us about quark-lepton unification ?

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☞ **Neutrino experiments have told us a great deal about neutrino masses and mixings**

☞ **They have posed two major puzzles for theory:**

☞ **(i) Why are $m_\nu \ll m_{u,d,\ell}$?**

☞ **(ii) Why two of the neutrino mixings are so much larger than the corresponding quark mixings ?**

☞ **Subject of this talk: Do they tell us anything about quark-lepton unification which is such an integral feature of the idea of grand unification ?**

I will argue that higher precision measurement of θ_{13} will throw light on this question.

Basic assumptions

☞ **(i) There are three generations of neutrinos and they are Majorana fermions.**

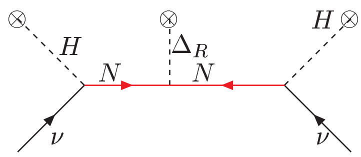
(ii) The smallness of neutrino masses is explained by the seesaw mechanism.

(iii) There is quark-lepton unification based on the Pati-Salam $SU(4)_c$ group or SO(10) at high scale.

(iv) The large atmospheric and solar mixings originate from the neutrino mass matrix.

An appealing explanation of small neutrino mass is the Seesaw mechanism:

☞ **Add N_R to the standard model**



☞ **Leads to** $M_\nu = -M_D^T M_R^{-1} M_D$**;**

since $M_R \gg M_D$**, explains why** $m_\nu \ll m_{e,u,d}$ **etc .**
(Type I seesaw)

☞ $M_\nu = M_L - M_D^T M_R^{-1} M_D$**;**
Type II: in theories with parity symmetry

Seesaw and quark-lepton symmetry

☞ **Quark-lepton symmetry was already suspected from similarity of weak interaction between quarks and leptons: seesaw mechanism strengthens this belief since it brings in the N_R to the fermion spectrum.**

☞

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$\begin{pmatrix} u_R \\ d_R \end{pmatrix} \qquad \begin{pmatrix} N_R \\ e_R \end{pmatrix}$$

☞ **It is tempting to arrange them (a la Pati-Salam) as**

$$\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_{L,R}.$$

Perhaps another hint in favor of Q-L unification

☞ $\Delta m^2_A \sim 2.5 \times 10^{-3}$ **eV^2 implies via seesaw** $\rightarrow$ $M_R \sim 10^{15}$ **GeV**
close to the grand unification scale of 10^{16} GeV

☞ **The minimal GUT group that unifies standard model fermions with the right handed neutrino is SO(10) which contains the quark-lepton unifying group $SU(4)_c$.**

☞ **Are there any tell-tale predictions for mixings that can distinguish between SO(10) or $SU(4)_c$ vrs no such symmetry**

☞ **This question is important for the future direction of unification!**

Mixings

☞ **Three mixing angles and three phases:**
$sin^2\theta_\odot \simeq 0.28 - 0.37 \quad 3\sigma$; $\theta_{13} \leq 0.22$; $\theta_{23} \simeq 45^o \pm 8^0 \quad 3\sigma$;

Very different from quark mixings.

☞ **Is this difference an evidence against quark-lepton unification ? The answer is "No"; since seesaw mechanism introduces a new flavor structure for RH neutrinos completely unrelated to quarks .**
In fact, there are a number of ways to understand large neutrino mixings within quark-lepton unified framework.
So how can one tell from neutrino mixings whether Q-L unification is there or not ?

☞ **It is possible to make a case that if there is quark-lepton symmetry θ_{13} is likely to be not far below the present upper limit!!**

Neutrino mass matrix and neutrino mixing

☞ Near maximal θ_A and large $\theta_\odot$ for normal hierarchy

➢ Choose basis where e, μ, τ are mass eigenstates

➢ $$\mathcal{M}_\nu = \sqrt{\Delta m_A^2}\begin{pmatrix} d\epsilon & b\epsilon & a\epsilon \\ b\epsilon & 1+\epsilon & 1 \\ a\epsilon & 1 & 1+c\epsilon \end{pmatrix}; n \geq 1.$$

➢ $(\epsilon \approx \sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}})$

➢ Leads to both large θ_{23} and large θ_{12} and small θ_{13}.

☞ **How to understand this mass matrix?**

1. (i) With quark-lepton unification or

2. (ii) using only leptonic symmetries

☞ **Strategy:**

(i) I describe three realistic models with $SU(4)_c$ and all of then lead to "large" θ_{13}

(ii) What does it take to get a naturally tiny θ_{13}- answer will be "an exclusive leptonic symmetry which the quarks cannot share."

Example (i): SO(10)

Babu, RNM (1992); Bajc, Senjanovic, Vissani (2002); Goh, RNM, Ng (2003)

☞ **SO(10) + 10 and 126 Higgs field**

☞

- Equations for fermion mass matrices
  $M_u = h < H_u > +f < \Delta_u >$
  $M_d = h < H_d > +f < \Delta_d >$
  $M_e = h < H_d > -3f < \Delta_d >$
  $M_{\nu^D} = h < H_u > -3f < \Delta_u >$
- It follows that
  $f \;=\; \frac{1}{4<\Delta_d>}(M_d - M_\ell)$
  (Relation valid at GUT scale)
- First important relation
  $M_\ell \;=\; c_u M_u + c_d M_d$

☞ Neutrino mass from type II seesaw

- ➢ Seesaw formula in SO(10)
  $\mathcal{M}_\nu \simeq f\frac{v_{wk}^2}{v_R} - M_{\nu^D} f {v_R}^{-1} M_{\nu^D}$; (Type II seesaw)
- ➢ Suppose the first term dominates →
  $\mathcal{M}_\nu \simeq \frac{v_{wk}^2}{4v_R<\Delta_d>}(M_d - M_\ell) \equiv c(M_d - M_\ell) \quad \cdots \quad$ [1]
  $c \;\sim\; 10^{-10}$
- ➢ $M_\ell \;=\; c_u M_u + c_d M_d$: $\cdots$ [2]
  Implies
- ➢ These are the key formulae for understanding large mixings
- ➢ (2) → $U_{PMNS} \simeq U_\nu$
- ➢ (1) → neutrino mixings are large. (next page)

Large mixings from minimal SO(10)

☞

➢ At seesaw scale $M_\nu = 10^{-9}(M_d - M_\ell) \rightarrow$

$$\mathcal{M}_\nu = c m_b \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^3 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & (1 - m_\tau/m_b) \end{pmatrix}$$

$\lambda \simeq 0.22$

➢ Important point to note is that at the GUT scale $m_b \simeq m_\tau(1 + O(\lambda^2))$

➢ $b - \tau$ mass convergence implies

$$\mathcal{M}_\nu = m_b c \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^3 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix} = m_b c \lambda^2 \begin{pmatrix} \lambda^2 & \lambda^2 & \lambda \\ \lambda^2 & 1 + \lambda & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

☞ $\theta_{13} \sim \lambda$ **(actual prediction** ~ 0.18**)**

$\theta_{13} \equiv U_{e3}$ in several SO(10) models with 126

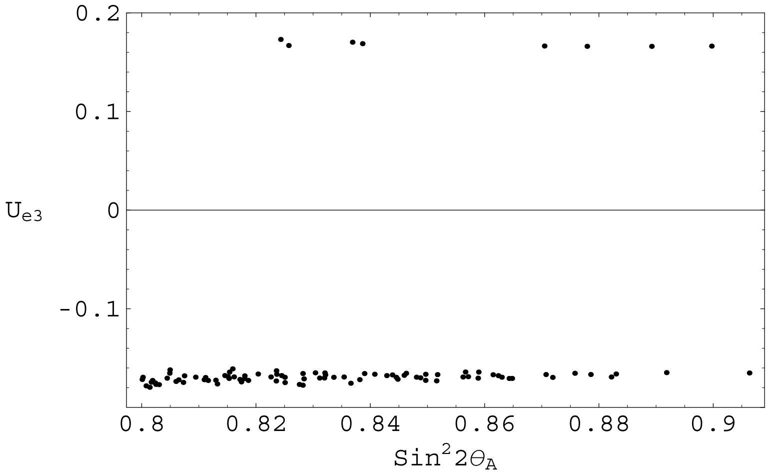


Figure 1: $U_{e3} \equiv \theta_{13}$ and just below the present upper limit: "high" value due to quark lepton symmetry but no $\mu \leftrightarrow \tau$ symmetry (see before)

☞ Add 120 to get CKM CP violation; still predictive for U_{e3}:

Dutta, Mimura, R. N. M.; Bertolini, Frigerio, Malinsky; Wang and Yang

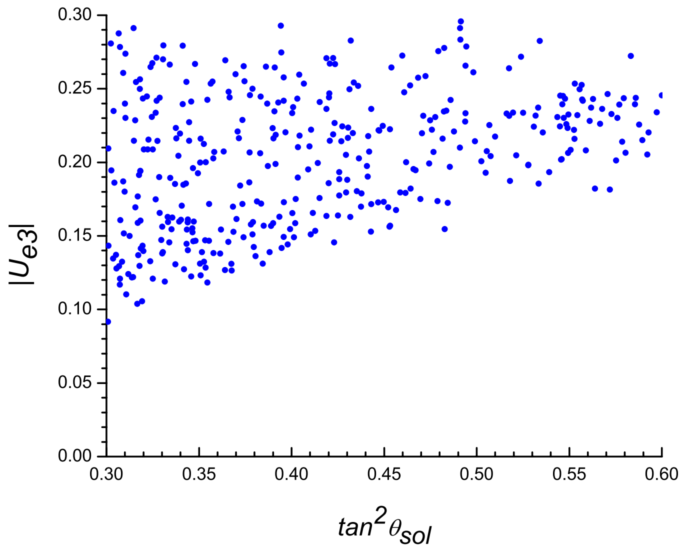


Figure 2: scatter corresponds to different allowed quark mass values

Example (ii): $SU(4)_c$ plus type II seesaw

☞ Radiative magnification for degenerate neutrinos

- ➢ Assume type II seesaw and $SU(4)_c$:
  $M_\nu \;=\; f v_L - M_D^T (f v_R)_R^{-1} M_D$;
- ➢ High scale Quark-lepton Unification $\rightarrow M_D \simeq M_q$ and suppose $f = I$;
- ➢ At seesaw scale $m_1 \simeq m_2 \simeq m_3$ and $V^q_{ij} = U^\ell_{ij}$;
- ➢ Predicts $\frac{\Delta m^2_\odot}{\Delta m^2_A} \sim \frac{m_c}{m_t} \approx 10^{-2}$
- ➢ extrapolate $\rightarrow$ Both V_{us} and V_{cb} get magnified to large values and θ_{13} remains within present limits i.e. $\theta_{13} \sim 0.1$.
- ➢ Requires $m_{ee} \geq 0.1$ eV; Testable in $\beta\beta_{0\nu}$ expts. In the range reported by Klapdor-Kleingrothaus et. al.

Parida, Rajasekaran and RNM, 2003

Evolution of mixing angles

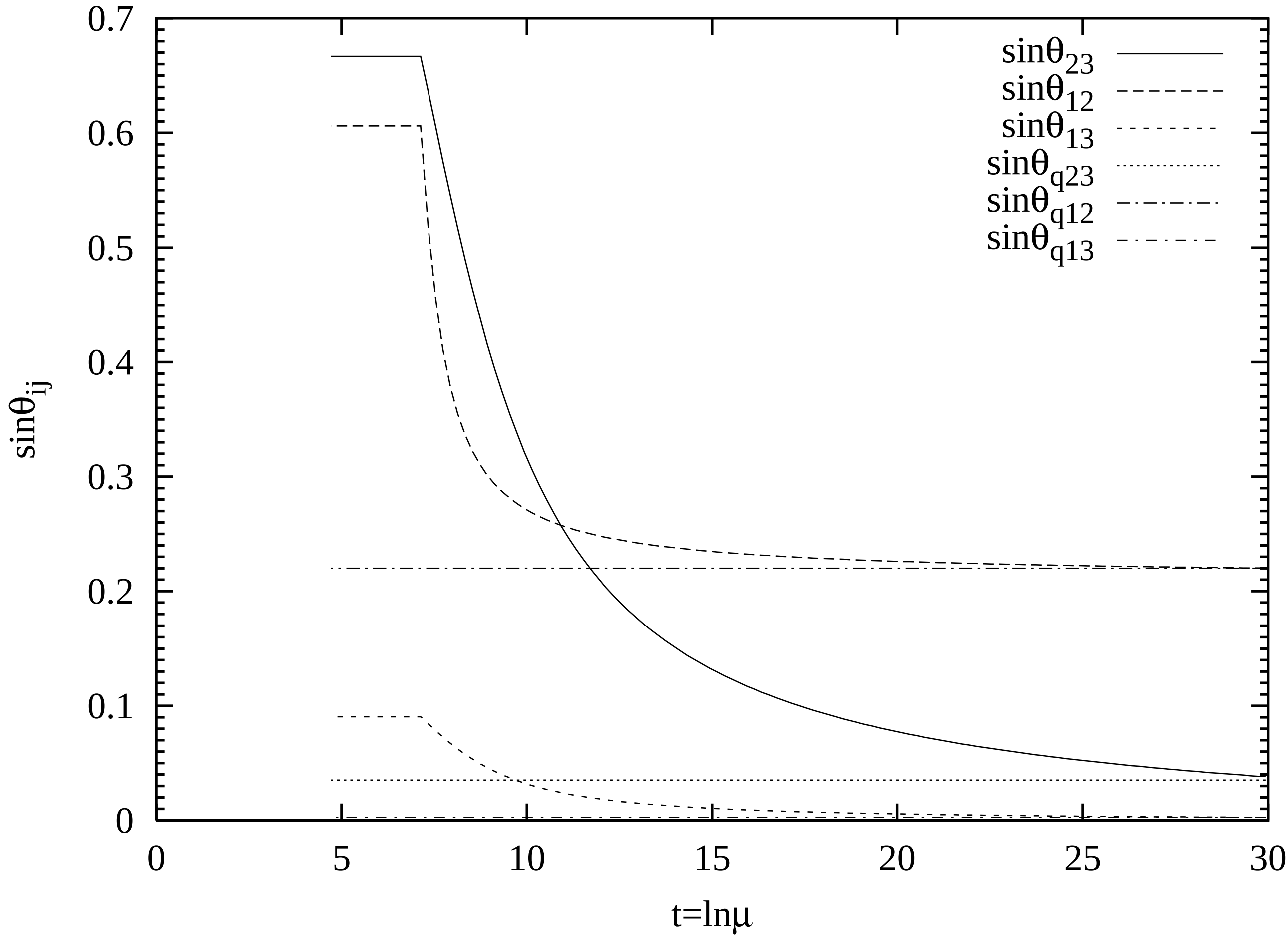


Figure 3: High scale mixing unification and Radiative magnification of mixing angles for degenerate neutrinos

☞ $\theta_{13} \simeq 0.08$.

Solar angle and quark-lepton unification

☞ **Inspired by the observation that $\theta_\odot + \theta_C \simeq 45^o$;**

Called Quark-Lepton complementarity

Raidal; Minakata and Smirnov

☞ **Suppose tree level $\theta_{12} = 45^o$ and $\theta_C = 0$; since $U_\ell^\dagger U_\nu = U_{PMNS}$, then if $U_\ell = U_{CKM}$, then departure of solar mixing angle from 45^o is related to Cabibbo angle:**

$U_\ell = U_{CKM}$ **is a signal of $SU(4)_c$.**

Is it possible to have such a theory?

$SU(4)_c$ and Quark-lepton complementarity

Frampton and R. N. M., 2004

☞ **Start with gauge theory based on $SU(2)_L \times SU(2)_R \times SU(4)_c$ and $U(1)_X$ where $X = F_1 - F_2 - F_3$ (behaves like $L_e - L_\mu - L_\tau$)**

Leading order: Inverted hierarchy and exact bimaximal mixing i.e. $U_{PMNS} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

☞ **Also one has $M_u = M_{\nu^D} = tan\beta M_d = tan\beta M_\ell$ so that at tree level $U_{CKM} = I$ or $\theta_C = 0$.**

☞ **Use leading order corrections $\rightarrow$**

$\theta_\ell \simeq \frac{\theta_C m_s}{\Delta_{rad} m_\mu}$

$\theta_\ell \leq \theta_C$

Predictions of a QLC theory

☞ **Leads to QLC with $sin^2\theta_\odot \simeq 0.34$ within 2 σ of data.**

When other corrections are included, it can come closer to central value.

Ferrandis, Pakvasa; Kang, Kim, Lee

☞ **Predicts $\theta_{13} \simeq 0.1$**

A generic argument

☞ **Note:** $U_{PMNS} = U_\ell^\dagger U_\nu$;

Q-L symmetry $\rightarrow U_\ell = U_d \equiv U_{CKM}$;

i.e. $U_\ell = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$;

Suppose seesaw gives large mixing angle-i.e.

$$U_\nu = \begin{pmatrix} c & s & \epsilon \\ -\frac{s}{\sqrt{2}} & \frac{s}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{s}{\sqrt{2}} & \frac{s}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} (U_\ell^\dagger U_\nu)_{13} \sim \frac{\lambda}{\sqrt{2}};$$

"Large" θ_{13}.

☞ **The only exception is the class of lopsided models where large** θ_{23} **comes from the charged lepton sector**

Albright (hep-ph/0502161)

Q-L unification models seem to predict "large" θ_{13} i.e. $\theta_{13} \geq 0.1$. Suppose θ_{13} is very small i.e. ≤ 0.04 what does it mean ?

☞ **Will require extreme fine tuning in quark-lepton unified frameworks; very small θ_{13} therefore probably an evidence against quark-lepton unification !**

As many other cases of fine tuning, this may be an indication of a different underlying symmetry. One such symmetry is pure leptonic $\mu - \tau$ interchange symmetry.

R. N. M. hep-ph/0408186; Grimus, Joshipura, Kaneko, Lavoura, Sawanaka, Tanimoto; hep-ph/0408123

$\mu - \tau$ symmetry and tiny θ_{13}

☞ **Recall mass matrix for three generations:**

➢ $\mathcal{M}_\nu = \sqrt{\Delta m^2_A}\begin{pmatrix} d\epsilon & b\epsilon & a\epsilon \\ b\epsilon & 1+c\epsilon & 1 \\ d\epsilon & 1 & 1+\epsilon \end{pmatrix}; (a, b, c, d \sim 1)$

➢ $\epsilon \simeq \sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}}$ and $\theta_{13} \sim [(b-a)\epsilon + (c-1)\epsilon^2]$

➢ Note that if $a = b$ and $c = 1$, we have $\rightarrow \theta_{13} = 0$

➢ This is the exact $\mu \leftrightarrow \tau$ symmetric limit.

➢ Exact $\mu - \tau$ symmetry $\rightarrow \theta_{13} = 0$

How big is θ_{13} with broken $\mu - \tau$ symmetry?

☞ **Consider** $\mathcal{M}_\nu = \sqrt{\Delta m_A^2}\begin{pmatrix} d\epsilon & b\epsilon & b\epsilon \\ b\epsilon & 1+c\epsilon & 1 \\ b\epsilon & 1 & 1+\epsilon \end{pmatrix}$;

leads to $\epsilon \sim 4\sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}} F(b,d)$;

$\theta_{13} \simeq \frac{1}{4\sqrt{2}}\left(\frac{\Delta m_\odot^2}{\Delta m_A^2}\right) b(1-c) \simeq 0.04$

A truly small θ_{13} will be an indication of an underlying $\mu - \tau$ symmetry.

In this case,departure from maximality in the atmospheric mixing angle is correlated with θ_{13} i.e.
$\theta_A \simeq \frac{\pi}{4} - \frac{1-c}{4}\sqrt{\frac{\Delta m_\odot^2}{\Delta m_A^2}}$

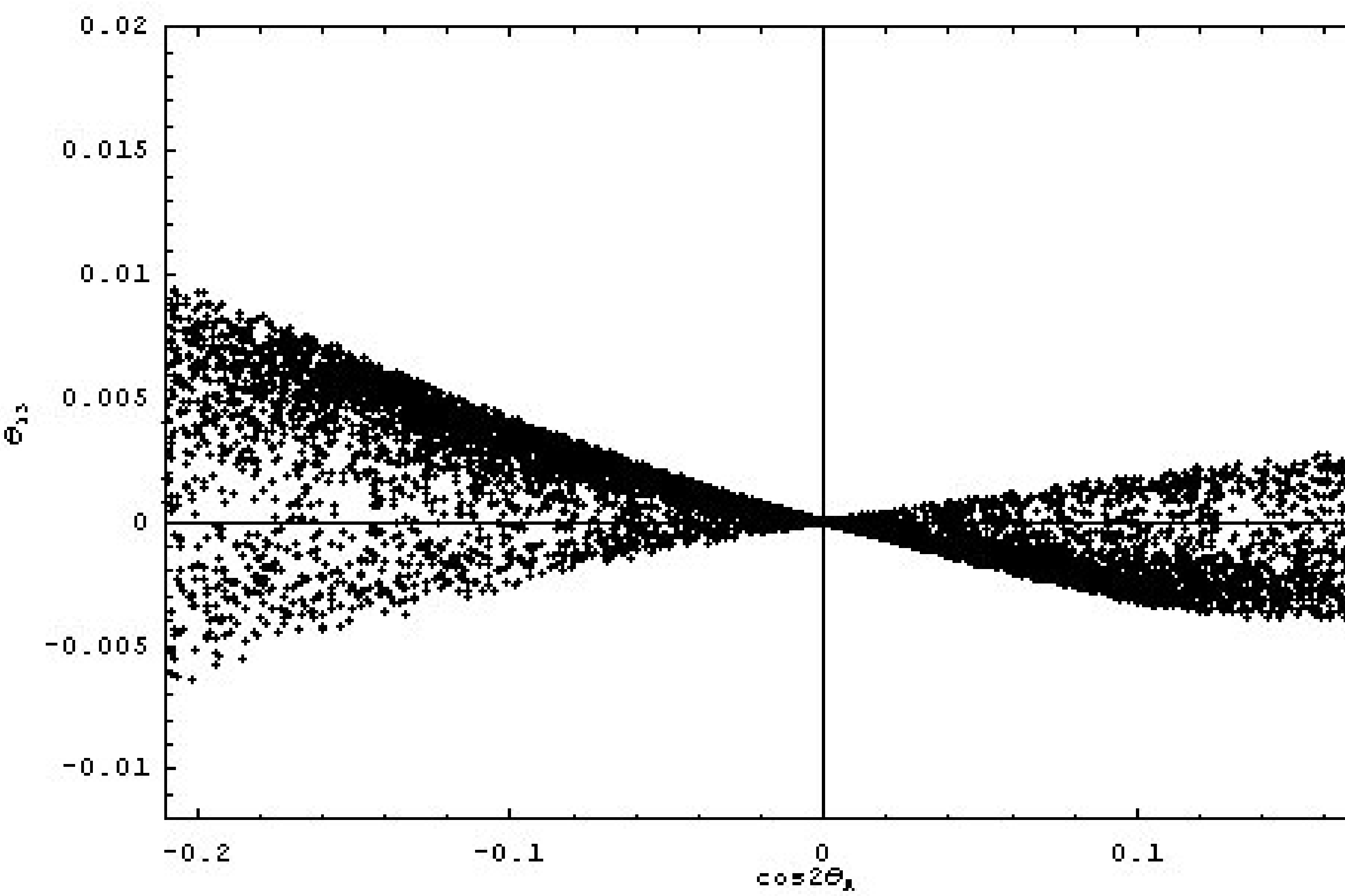
☞ **(i) Tiny θ_{13} i.e. (≤ 0.04) requires severe fine tuning in the Q-L unif. models- but very natural in $\mu - \tau$ symmetric models.**
(ii) A further signal of underlying $\mu - \tau$ symmetry is correlation between $\theta_A - \frac{\pi}{4}$ and θ_{13}. No such correlation expected in fine tuned Q-L symmetric models.
(iii) Hence the conclusion that very tiny θ_{13} will point away from quark lepton unification.

CONCLUSION

☞ **(i) A high precision measurement of θ_{13} is likely to provide a probe of whether there is quark lepton unification at high scale beyond the standard model.**

(ii) A value of $\theta_{13} \leq 0.04$ Plus a correlation of θ_{13} with $\theta_A - \pi/4$ will signal the existence of approximate $\mu - \tau$ symmetry ; both simultaneously not expected in Q-L unified models.

(iii) Value above ≥ 0.08 or so would be quite compatible quark-lepton unification though not a proof of it.


0.02
0.015
0.01
0.005
0
-0.005
-0.01
θ_{13}
-0.2
-0.1
0
0.1
$\cos 2\theta_A$