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# $\delta T/T$ , Neutrino Oscillations

## And Proton Decay

- Introduction / Motivation
- Models
  - $\delta T/T$  (inflation)
  - $\nu$  Oscillations
  - Baryon asymmetry
- Proton Decay

with Dvali, Lazarides, Kyae  
Senoguz, Schaefer, ...

PHYSICS BEYOND THE  
STANDARD MODEL (SM)  
REQUIRED BY :

• Neutrino Oscillations

$$(\Delta m_{SM}^2 \sim 10^{-10} \text{ eV}^2 \ll \Delta m_{ATM}^2, \Delta m_{\nu_e}^2)$$

$\uparrow$  dim 5

•  $\delta T/T$  (Inflation) ( $\sim 10^{-5}$ )

• Non-baryonic DM ( $\Omega_{CDM} \approx 0.25$ )

• Baryon Asymmetry ( $n_B/s \sim 10^{-10}$ )

Gauge(d)  $U(1)_{B-L}$  (SM: global  $U(1)_{B-L}$ )

- Anomaly cancellation  
⇒ 3 (right handed)  $\nu_R$

- See saw Mechanism

- Leptogenesis

? B-L breaking scale

# Hierarchy Problem(s)

- $M_W \ll M_{P(\text{Planck})}$
- $M_{B-L} \ll M_P$  ( $\nu$  Osc)
- $f_a(\text{axion}) \sim 10^{10} - 10^{12} \text{ GeV} (\ll M_P)$   
↑  
CDM
- $M_{\text{inflaton}} \ll M_{\text{GUT}} (\delta\rho/\rho, \text{etc.})$

Supersymmetry can certainly help here, especially if the SUSY breaking scale (in the observable sector) is of order TeV.

⇒ starting point

could be MSSM  $\times$

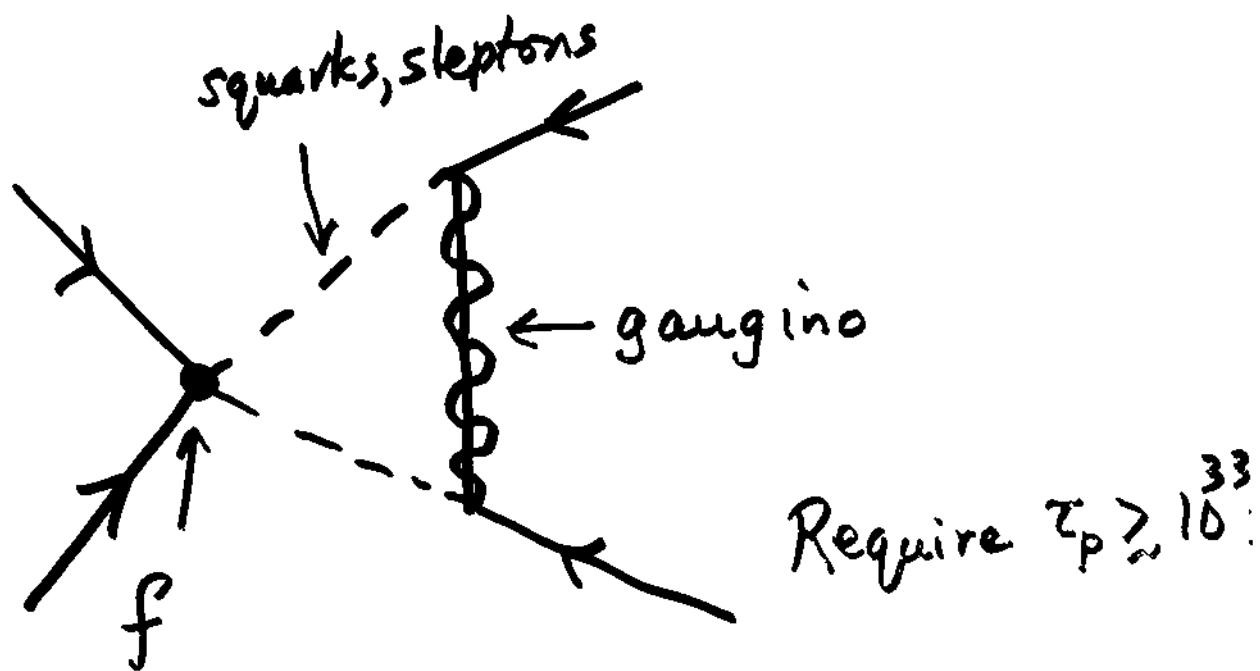
$U(1)_{B-L}$

- $Z_2$  'matter' parity eliminates rapid proton decay and delivers cold dark matter (LSP).

- But dim 5 p decay is still problematic because <sup>superpotential</sup> of  $\lambda$  terms such as

$$\frac{f}{M_P} Q Q Q L \quad (\text{allowed by } U(1)_{B-L})$$

$$\left\{ \text{Cf: } \frac{1}{M_P^2} q q q l \text{ in SM } \Rightarrow \tau_p \geq 10^{42} \text{ yr} \right\}$$



⇒ Coefficient  $f \lesssim 10^{-7}$  (how?)

One solution:

$$\mathbb{Z}_2 \subset U(1)_R$$

Under R: Q, L carry charge  $\frac{1}{2}$   
& W (superpotential)  
has unit charge

( $\mathcal{L}(x) \rightarrow \int d^2\theta W$  invariant)

Working with MSSM  $\times$

$$U(1)_{B-L} \times U(1)_R$$

one can

- easily realize inflation;
- determine  $M_{B-L}$  (from  $\delta T/T$   
 $\hookrightarrow$  not possible from gauge coupl unification
- put bounds on right handed  $\nu$  masses;
- resolve  $\mu$  problem;
- explain  $n_b/s$  via leptogenesis.



Starting theory quite simple

$$W = W_{\text{MSSM}} + W_{\text{INF}} + W_{\text{RH}}$$

$$W_{\text{INF}} = \kappa S (\bar{\phi} \phi - M_{\text{B-L}}^2)$$

Unique form at renormalizable level. Higher order terms don't affect conclusions regarding  $\delta T/T$ ,  $M_{\text{B-L}}$ .

# Inflation (101) (inflation linked to some phase transition)

Recall that we deal with spontaneously broken gauge theories:

Example:  $U(1)_{B-L} \xrightarrow{\langle \phi \rangle} 1$

e.g. Superconductor

Consider the potential energy density

$$V \sim \kappa^2 |\phi^* \phi - M_{B-L}^2|^2 + 2\kappa^2 S^2 / \phi$$

Minimum corresponds to

$$|\phi| = M_{B-L}, \quad \langle S \rangle = 0 \quad (\Rightarrow V = 0)$$

$\Rightarrow U(1)$  spontaneously broken (today)

To realize an inflationary epoch, <sup>some</sup> fields must be displaced from their present positions.

Think of  $S$  as temperature ( $\approx 0$  at present).

For  $S \gg M_{B-L}$  (very early universe) we should have  $|\phi| = 0$ , so that

$$V \sim \kappa^2 M_{B-L}^4$$

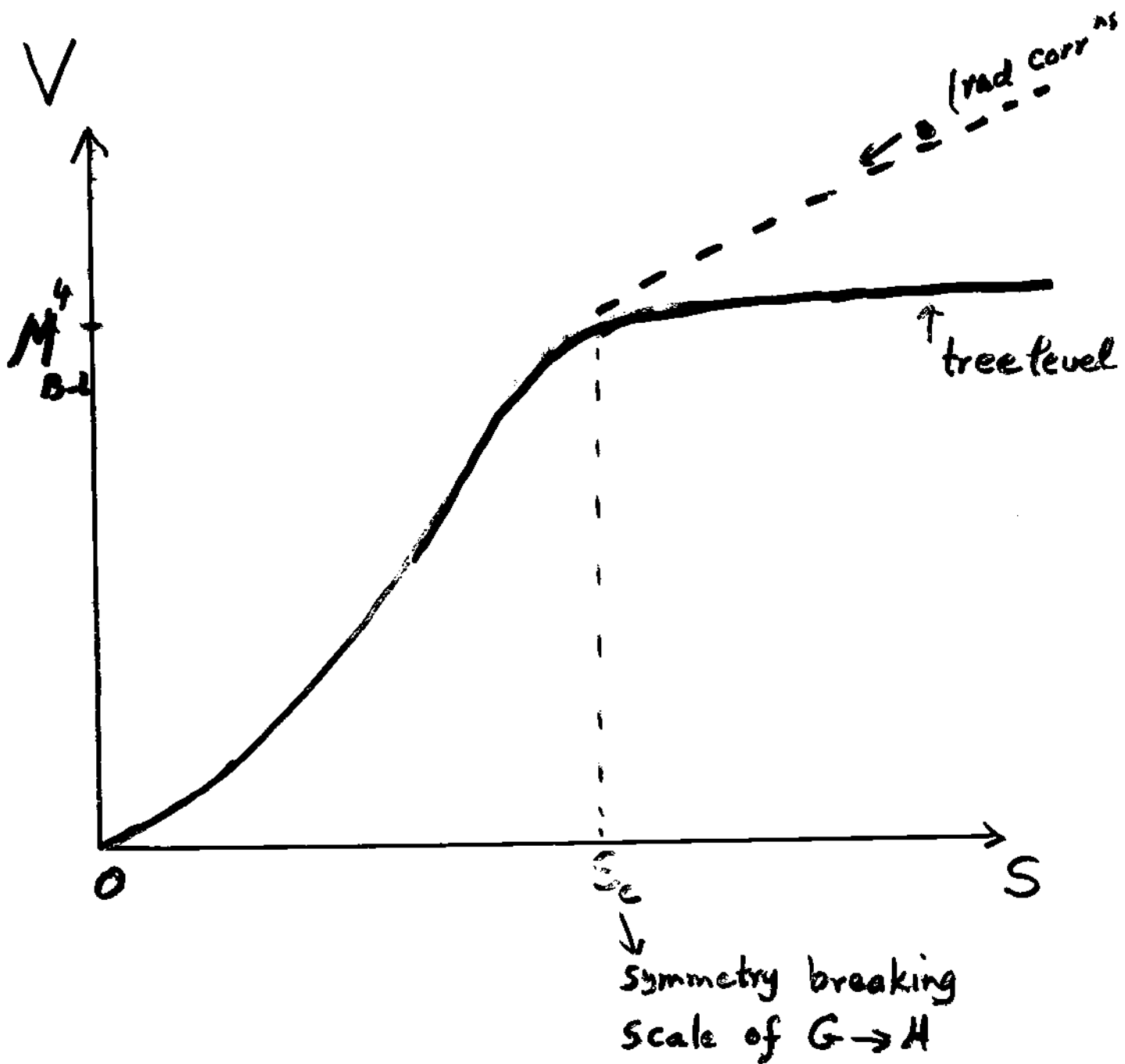
↑ symmetry restoration  
(Cf: super-conductors)

Thus, we expect to have gauge symmetry restoration above some 'critical' temperature  $S_c > M_{B-L}$ .

A non-zero vacuum energy density  $\Rightarrow$  exponential expansion of the universe  $\Rightarrow$  inflation.

? How does inflation terminate

Through quantum corrections that are calculable:



For  $S > S_c$ ,

$$V_{\text{eff}}(S) \sim \mu^4 \left[ 1 + \frac{\sqrt{K^2}}{32\pi^2} \ln\left(\frac{K^2 S^2}{\Lambda^2}\right) \right]$$

↑  
Use to calculate  
 $\delta T/T, n, S/T$  ( $n = 1 - \frac{1}{N_g}$ )  
 $\approx 0.98 \frac{1}{N_g}$

# Predictions

- $\delta T/T \propto (M_{B-L}/M_{\text{Planck}})^2$   
 $\Rightarrow M_{B-L} \sim 10^{16} \text{ GeV}$   
( $\sim M_{\text{GUT}}$ )
- $n_s \approx 1 - \frac{1}{N}$  ( $N \sim 60$  e-foldings)  
 $\approx 0.98 - 0.99$
- $r \equiv T/S \ll 1$
- Lepto/Baryo-genesis /  $\nu$  Oscilla

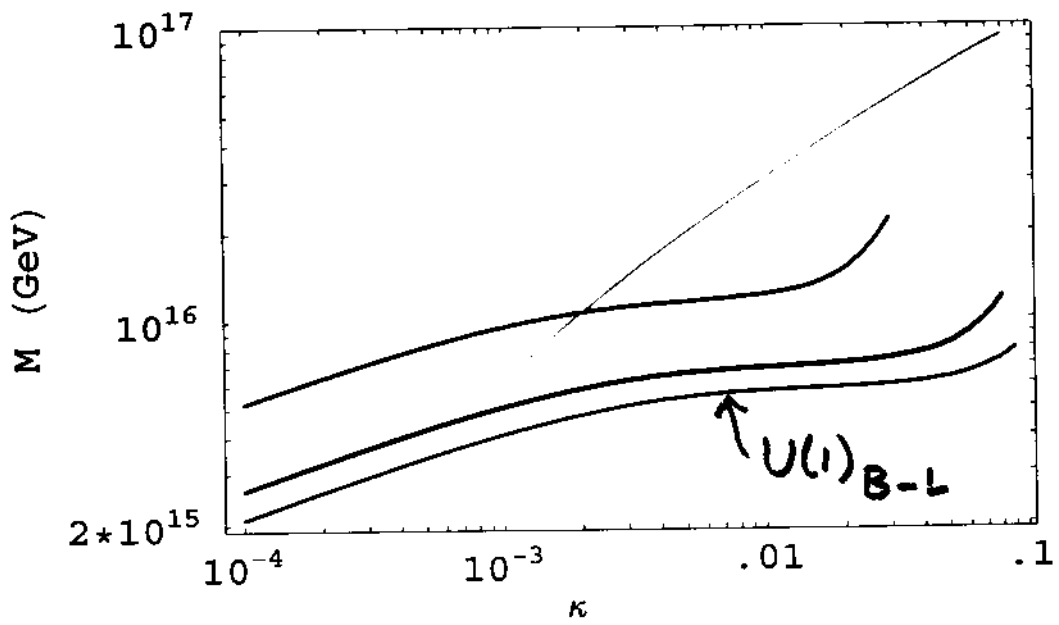


FIG. 2: The symmetry breaking scale  $M$  vs.  $\kappa$ , for  $\mathcal{N} = 1$  (green),  $\mathcal{N} = 2$  (blue),  $\mathcal{N} = 16$  (purple), and for shifted hybrid inflation (orange).

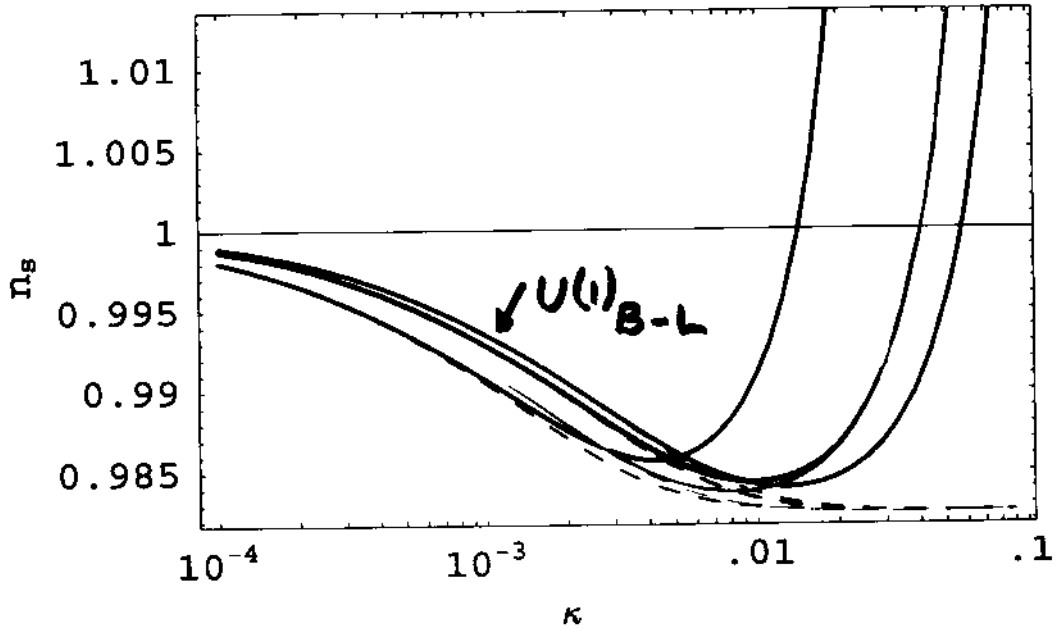


FIG. 1: The spectral index  $n_s$ , vs.  $\kappa$ , for  $\mathcal{N} = 1$  (green),  $\mathcal{N} = 2$  (blue),  $\mathcal{N} = 16$  (purple), and for shifted hybrid inflation (orange). Solid lines - with SUGRA correction, dashed lines - without SUGRA correction.

Note:  $\kappa \simeq \frac{\text{inflaton mass}}{M} \lesssim 10^{-2}$   
↑  
(from  $T_{\text{reheat}}$ )



Table 1 Power Law  $\Lambda$ CDM Model Parameters- WMAP Data Only

Parameter		Mean (68% confidence range)	Maximum Likelihood
Baryon Density	$\Omega_b h^2$	$0.024 \pm 0.001$	0.023
Matter Density	$\Omega_m h^2$	$0.14 \pm 0.02$	0.15
Hubble Constant	$h$	$0.72 \pm 0.05$	0.68
Amplitude	$A$	$0.9 \pm 0.1$	0.80
Optical Depth	$\tau$	$0.166^{+0.076}_{-0.071}$	0.11
Spectral Index	$n_s$	$0.99 \pm 0.04$	0.97
	$\chi^2_{eff}/\nu$		1431/1342

\*Fit to WMAP data only

Table 2. Derived Cosmological Parameters

Parameter	Mean (68% confidence range)
Amplitude of Galaxy Fluctuations	$\sigma_8 = 0.9 \pm 0.1$
Characteristic Amplitude of Velocity Fluctuations	$\sigma_8 \Omega_m^{0.6} = 0.44 \pm 0.10$
Baryon Density/Critical Density	$\Omega_b = 0.047 \pm 0.006$
Matter Density/Critical Density	$\Omega_m = 0.29 \pm 0.07$
Age of the Universe	$t_0 = 13.4 \pm 0.3$ Gyr
Redshift of Reionization <sup>b</sup>	$z_r = 17 \pm 5$
Redshift at Decoupling	$z_{dec} = 1088^{+1}_{-2}$
Age of the Universe at Decoupling	$t_{dec} = 372 \pm 14$ kyr
Thickness of Surface of Last Scatter	$\Delta z_{dec} = 194 \pm 2$
Thickness of Surface of Last Scatter	$\Delta t_{dec} = 115 \pm 5$ kyr
Redshift at Matter/Radiation Equality	$z_{eq} = 3454^{+385}_{-392}$
Sound Horizon at Decoupling	$r_s = 144 \pm 4$ Mpc
Angular Diameter Distance to the Decoupling Surface	$d_A = 13.7 \pm 0.5$ Gpc
Acoustic Angular Scale <sup>c</sup>	$\ell_A = 299 \pm 2$
Current Density of Baryons	$n_b = (2.7 \pm 0.1) \times 10^{-7} \text{ cm}^{-3}$
Baryon/Photon Ratio	$\eta = (6.5^{+0.4}_{-0.3}) \times 10^{-10}$

\*Fit to the WMAP data only

<sup>b</sup>Assumes ionization fraction,  $x_e = 1$

<sup>c</sup> $\ell_A = \pi d_A / r_s$

# • $\mu$ Problem

Consider the Superpotential

$$W = S \left( \underbrace{\kappa \bar{\phi} \phi}_{\substack{\text{breaks} \\ U(1)_{B-L}}} + \lambda H_u H_d - M_{B-L}^2 \right)$$

- SUSY unbroken with  $|\langle \phi \rangle| = |\langle \bar{\phi} \rangle| \neq 0$   
 $\langle H_u \rangle, \langle H_d \rangle$  and  $\langle S \rangle$ :
- After SUSY breaking,  $S$  acquires VEV  $\propto m_{3/2}$   
 $\Rightarrow \mu$  term  $\propto m_{3/2}$ .
- P decay: 'Matter' fields have R-charge =  $\frac{1}{2}$   
Thus,  $QQQL$  is eliminated.  
Note:  $\mathbb{Z}_2$  subgroup of  $U(1)_R$  acts as 'matter' parity.

# $T_{\text{reheat}} (T_r)$ / Leptogenesis

- Inflaton consists of  $S$  and  $\phi, \bar{\phi}$  fields;

- Relevant couplings include

$$\left\{ \begin{array}{l} N_i N_j \bar{\phi} \bar{\phi} \text{ (masses for right handed } \nu \text{)} \\ S H_u H_d \leftarrow \text{suppose this is absent} \\ S \bar{\phi} \phi \end{array} \right.$$

then

inflaton decays into right handed  $\nu$  &  $\tilde{\nu}$ .

If  $S H_u H_d$  present then  $T_r$  is somewhat higher.

Then

$$T_\gamma \sim \left( \frac{1}{10} - \frac{1}{50} \right) M_i$$

$(2M_i \leq m_i)$

Thus, we require at least one  $M_i \lesssim 10^{10} - 10^{11}$  GeV.

Suppose  $M_3 \sim 10^{14}$  GeV ( $16_3 16_3 \bar{16} \bar{16}$ )  
so that  $m_3 \sim m_D^2 / M_3 \sim 10^{-1}$  eV ( $\Delta m_{A1}$ )

Then, we could have

$$T_\gamma \sim 10^9 \text{ GeV}$$

$$m_\phi \sim 10^{12} - 10^{13} \text{ GeV}$$

$$M_1, M_2 \sim 10^{10} - 10^{11} \text{ GeV}$$

$\Rightarrow$  non-thermal leptogenesis possible.

With  $T_r \ll M_1, M_2 \ll M_3$ ,

$$n_L/s \lesssim 3 \times 10^{-10} \left( \frac{T_r}{m_\phi} \right) \left( \frac{M_i}{10^6 \text{ GeV}} \right) \left( \frac{m_{\nu_3}}{0.05} \right)$$

$$\Rightarrow T_r \gtrsim 2 \times 10^7 \text{ GeV} \left( \frac{10^{16} \text{ GeV}}{M} \right)^{1/2} \left( \frac{m_\phi}{10^{11} \text{ GeV}} \right)^{3/4} \left( \frac{.05 \text{ eV}}{m_{\nu_3}} \right)$$

For the simplest models,  $T_r \gtrsim 3 \times 10^7 \text{ GeV}$

Note: For quasi-degenerate right handed neutrinos, the lepton asymmetry  $\epsilon$  per neutrino decay can be of order unity, so that  $T_r \gtrsim 10^2 \text{ GeV}$  (TeV

$$\boxed{n_L/s \sim \frac{T_r \epsilon}{m_\phi}}$$

scale leptogenesis)

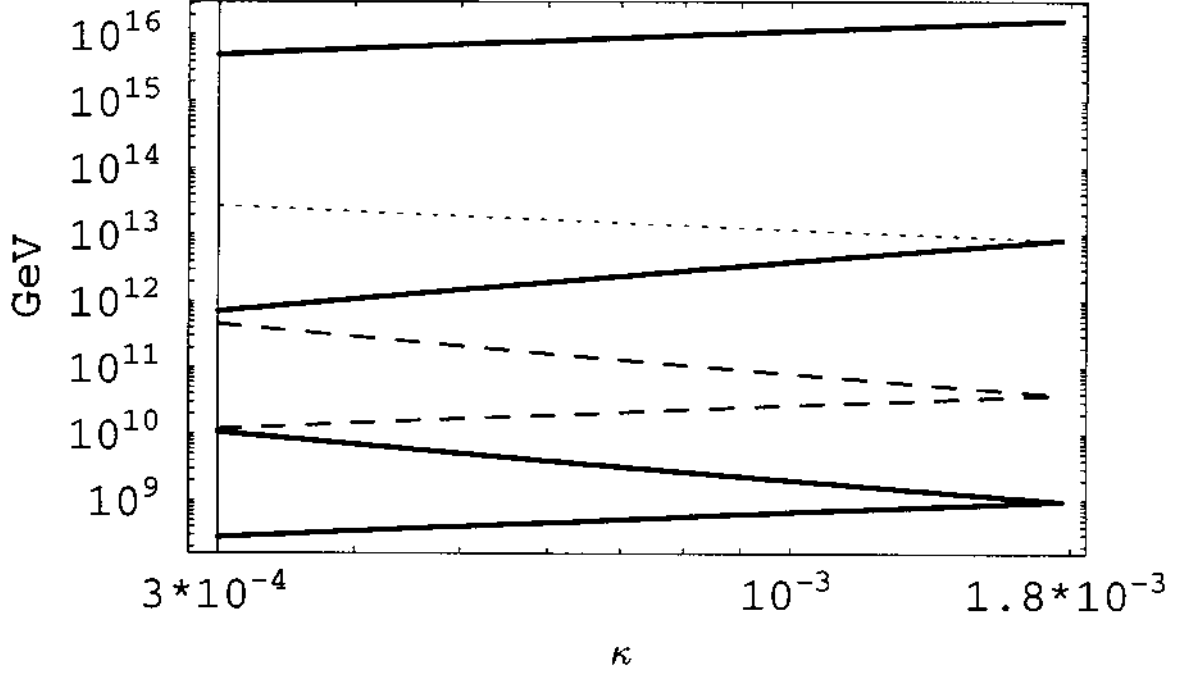


Figure 6: From bottom to top,  $T_r$ ,  $M_2$  (dashed lines),  $m_\chi/2$ ,  $M_3$  (dotted lines) and  $M$  as functions of  $\kappa$ , for shifted hybrid inflation with  $G = G_{PS}$  and degenerate left handed Majorana neutrinos. The regions for  $T_r$ ,  $M_2$  and  $M_3$  are bound by the baryon asymmetry and near maximal atmospheric mixing ( $\sin^2 2\theta_{23} \geq 0.95$ ) constraints. Note that  $M_3$  is bound below by  $m_\chi/2$ , and  $\kappa > 3 \times 10^{-4}$  is required for the inflationary trajectory for  $\beta = 0.5$ .

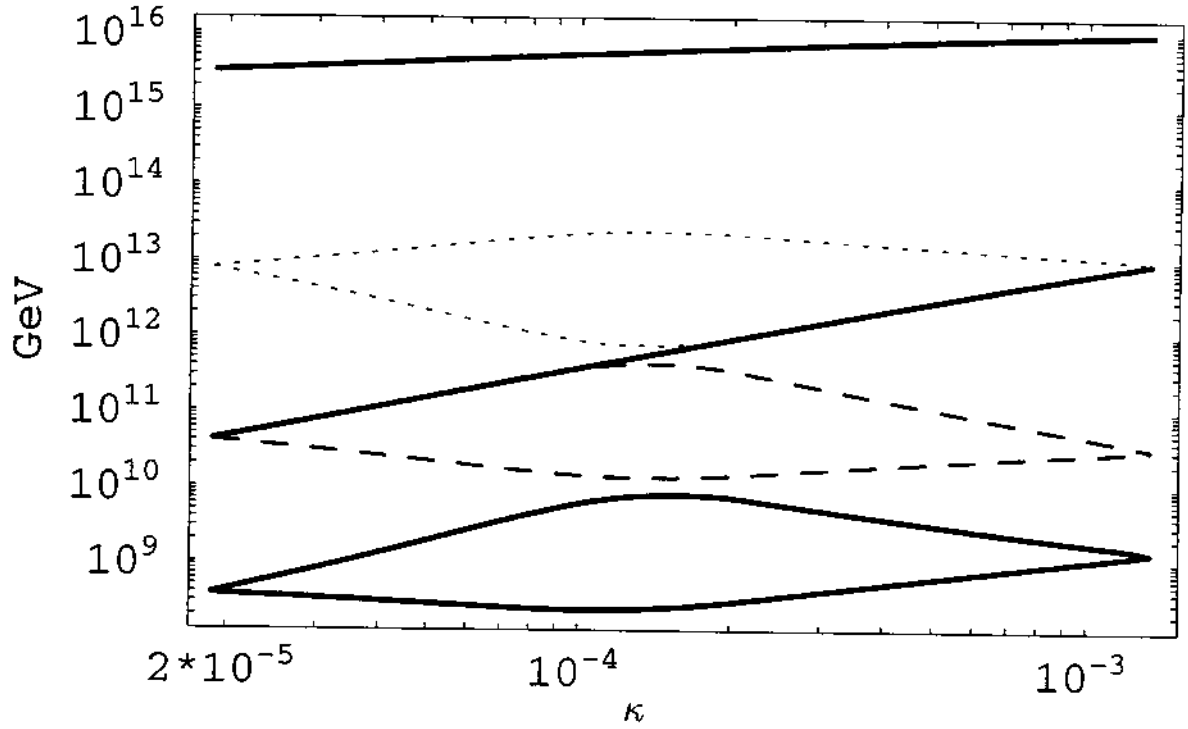


Figure 4: From bottom to top,  $T_\tau$ ,  $M_2$  (dashed lines),  $m_\chi/2$ ,  $M_3$  (dotted lines) and  $M$  as functions of  $\kappa$ , for SUSY hybrid inflation with  $G = SO(10)$  and degenerate left handed Majorana neutrinos. The regions for  $T_\tau$ ,  $M_2$  and  $M_3$  are bound by the baryon asymmetry and near maximal atmospheric mixing ( $\sin^2 2\theta_{23} \geq 0.95$ ) constraints.

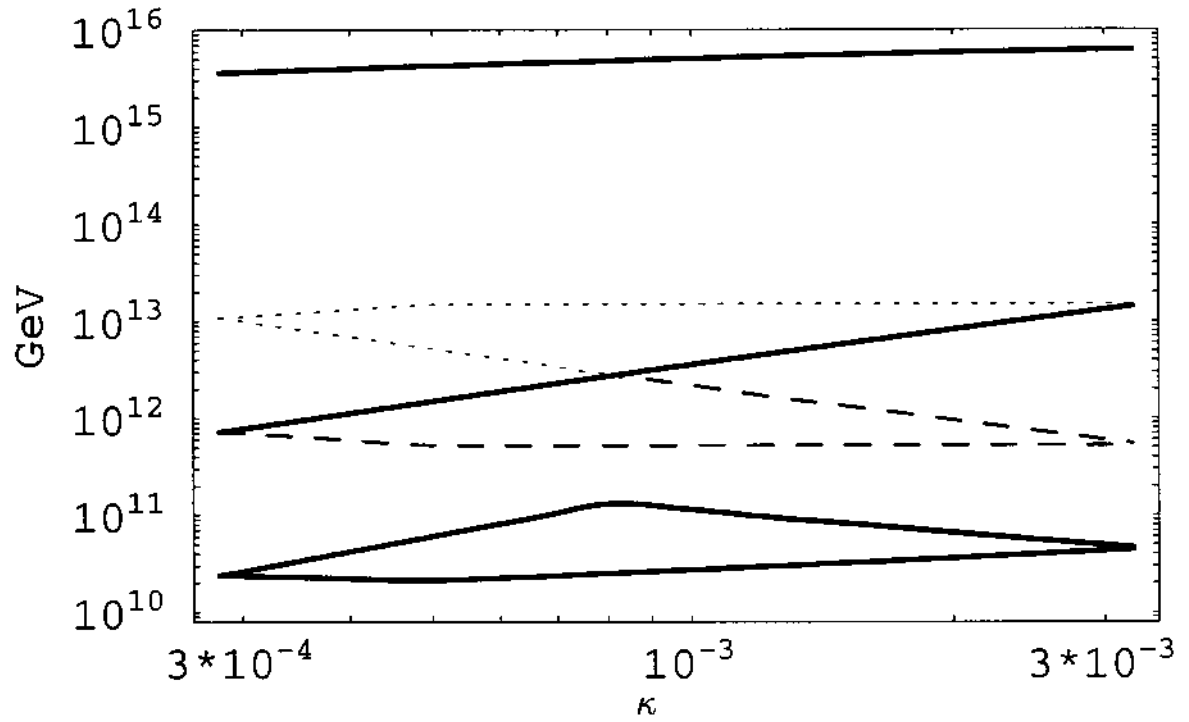


Figure 3: From bottom to top,  $T_\tau$ ,  $M_2$  (dashed lines),  $m_\chi/2$ ,  $M_3$  (dotted lines) and  $M$  as functions of  $\kappa$ , for degenerate left handed Majorana neutrinos. The regions for  $T_\tau$ ,  $M_2$  and  $M_3$  are bound by the baryon asymmetry and near maximal atmospheric mixing ( $\sin^2 2\theta_{23} \geq 0.95$ ) constraints. Note that the allowed regions for  $M_2$  and  $M_3$  are also constrained by  $M_2 \leq M_\chi/2 < M_3$ .



With  $G = \text{MSSM} \times U(1)_{B-L}$   
( $\times U(1)_R$ ):

- Proton is stable;
- Electric charge quantization is unexplained;
- Quark - Leptons not unified.  
(Gauge coupling unification)

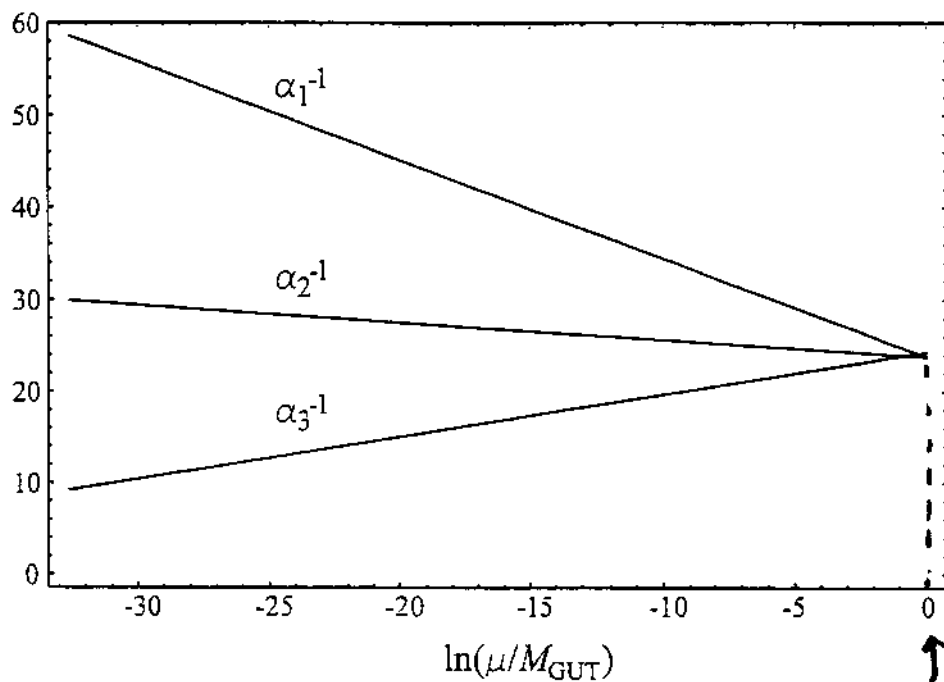
Obvious Generalization:

$$G = SO(10) \supset SU(4)_c \times SU(2)_L \times SU(2)_R$$

or  $G = E_6 \supset SU(3)_c \times SU(3)_L \times SU(3)_R$

# Evolution of Gauge Couplings

$$\alpha_i \equiv g_i^2 / 4\pi$$



• Unification of couplings  $\Rightarrow$

• Charge Quantization  
( $\Rightarrow$  monopoles)

• Quark-Lepton Unification  
 $\Rightarrow$  Proton Decay

$\uparrow$   
Unified Theory  
(single gauge coupling)

$$\underline{G = SU(4)_c \times SU(2)_L \times SU(2)_R}$$

Part. & Scalar  
⋮

- Quark - lepton unification

$$\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_L \quad \begin{pmatrix} u^c & u^c & u^c & \nu^c \\ d^c & d^c & d^c & e^c \end{pmatrix}_R$$

- Charge Quantization (monopole carries two Dirac quanta)

- Gauge bosons do not mediate proton decay

- Doublet - Triplet problem absent (huge advantage over SO(10))

Proton Decay?

# Minimal (higgs) Model

King, Shaf

- $\Phi(\bar{4}, 1, 2), \bar{\Phi}(4, 1, 2)$  break  $G_{422} \rightarrow M$
- $h(1, 2, 2) \leftarrow$  EW doublets
- $\mu$ -problem solved as before.

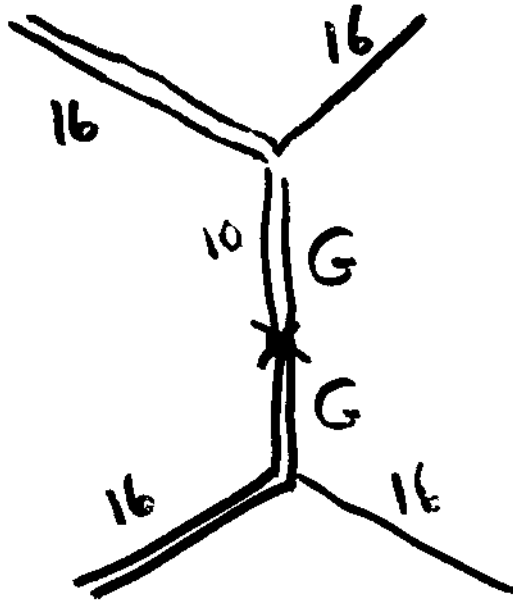
$$W \supset S(\kappa \bar{\Phi} \Phi + \lambda h^2 - \kappa M^2)$$

$$+ G \bar{\Phi} \bar{\Phi} + G \Phi \Phi$$

$\uparrow$   
(6, 1, 1)  $\leftarrow$  in principle,  
these mediate  
p decay  
(required to realise  
MSSM spectrum  
at low energies)

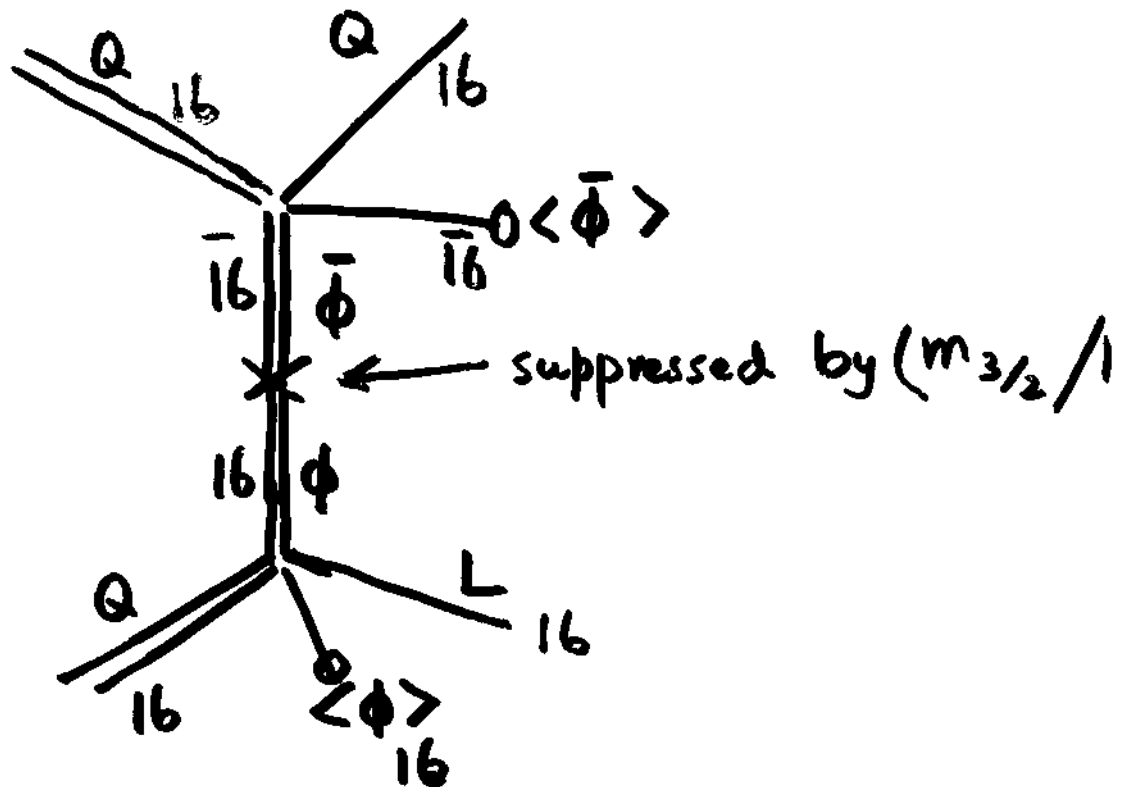
# Dimension Five Nucleon Decay

(i)



Allowed by  $G_{422}$ , but absent due to  $U(1)_R$ .

(ii)



Proton decay may arise through  
loop diagrams using dim 5  
operators:

$$\frac{1}{M_*} F^c F^c \phi \phi (\Rightarrow u_\phi^c d_\phi^c u^c e^c)$$

$$\frac{1}{M_*} F F \phi \phi (\Rightarrow u_\phi^c d_\phi^c u d)$$

(Add'l suppression from flavor  
symmetries.)

$$\tau_{p \rightarrow K \nu} \gtrsim 10^{38} \text{ yr}$$

# More Elaborate 4-2-2 Model

(including quark-lepton masses,  
mixing  
Mackawa,

- This means a non-minimal Higgs sector • In  $SO(10)$  notation:

	Higgs	Matter
$\underline{16}$	$C, C'$	$\psi_1, \psi_2, \psi_3$

$\overline{16}$	$\bar{C}, \bar{C}'$
-----------------	---------------------

$\underline{10}$	$H, H'$	$T$
------------------	---------	-----

$\underline{1}$	$\theta, Z, \bar{Z}, S$
-----------------	-------------------------

- Fields now also carry anomalous  $U(1)$  charge

# $U(1)_A$ charges (4 - 2 - 2)

	HIGGS	SCALARS (no vev)	
$\tilde{16}$	$C(-3)$ ↙	$C'(3)$ ↙	
$\overline{\tilde{16}}$	$\bar{C}(1)$	$\bar{C}'(5)$	
$\tilde{10}$	$H(-3)$	$H'(3)$	
$\tilde{1}$	$Z(-1)$	$\bar{Z}(-1)$	$S(2)$

## MATTER

$\tilde{16}_1$	$9/2$
$\tilde{16}_2$	$7/2$
$\tilde{16}_3$	$3/2$
$T(\tilde{10})$	$3/2$



By assigning suitable  $U(1)_A/U(1)_R$   
charges :

$$U_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$U_{MNS} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}$$

with  $\lambda^{0.5} \approx 0.5$

- dim 5 p decay is heavily suppressed with an estimated lifetime  $\sim 10^{39}$  yrs.

- Higher dimensional terms in the Kähler potential, if not suppressed by small coefficients, can induce dimension six proton decay

$\tau_{p \rightarrow e^+ \pi^0} \sim 10^{34-38}$  yrs  
if the cutoff is not far above the GUT scale.

# SO(10)

- B-L gauged ;
- $\nu^c \Rightarrow$  seesaw ;
- D-T splitting 'easier' (than SU(5))
- Two GUT scale vevs ;
- Matter parity  $\subset$  SO(10)  $\left\{ \begin{array}{l} \text{only} \\ \text{tensor fields} \\ \text{have vevs} \end{array} \right\}$

As an example consider

$$\begin{array}{ccc} 45_H & , & 16_H, \bar{16}_H \\ \downarrow & & \downarrow \\ 3-2_L-2_R-1 & & \text{SU}(5) \text{ (no monopoles} \\ \text{(monopoles)} & & \text{or strings)} \end{array}$$

# DW Mechanism

$$\langle 45_H \rangle = \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & a & & \\ & & & a & \\ & & & & a \end{pmatrix} \otimes i\tau_2$$

Then, from  $\begin{matrix} \downarrow & \text{10-plet} & \downarrow \\ T_1 & 45_H & T_2 \end{matrix}$ ,

doublets remain massless (finally one pair)  
triplets become superheavy.

But  $\bar{16}_H 45_H 16_H$  can destabilize  
the mechanism.

## To accomplish

- D-T splitting
- MSSM spectrum at low energies
- Inflation
- Complete absence of monopoles
- Absence of moduli,

One needs (it seems):

2 pairs of  $16, \bar{16}$  ;

2 45-plets ;

2 10-plets ;

Plus a bunch of singlets & add'l symmetries (such as  $U(1)_A$ ).

# One Solution (Barr, Raby)

$45_H$

$16_H, \bar{16}_H, \underbrace{16', \bar{16}'}$

$10_1, 10_2$

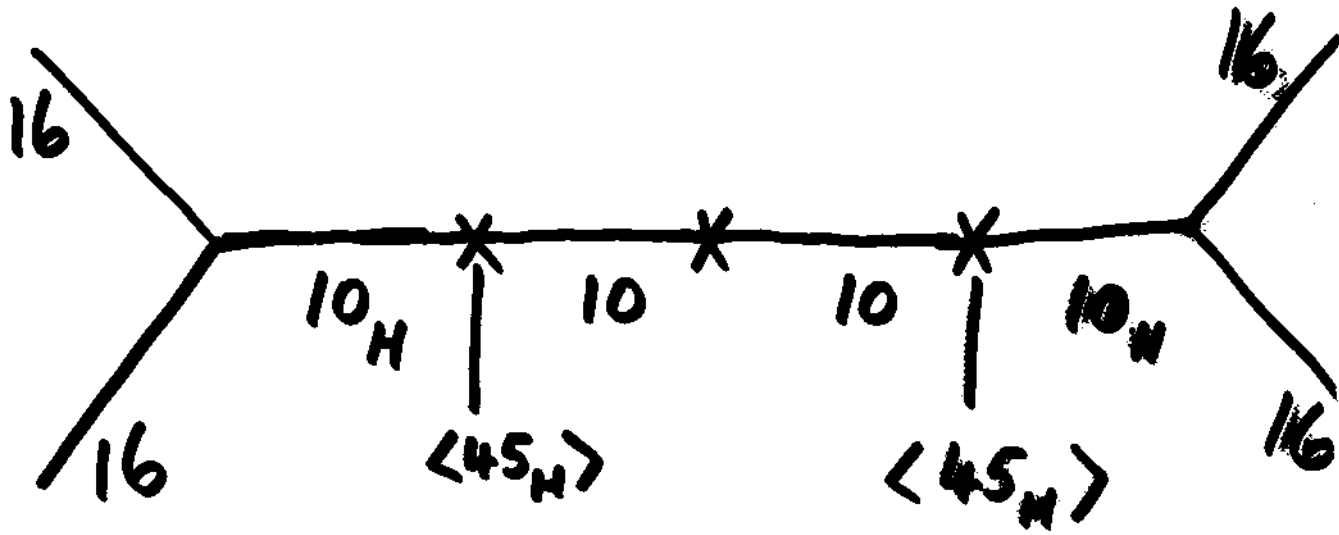
Zero vev along the  
SU(5) singlet dir<sup>n</sup>

+ bunch of singlets (avoid singlets  
with large vev.  
& small ( $\sim T_e$ )  
masses)

somewhat suppressed ;

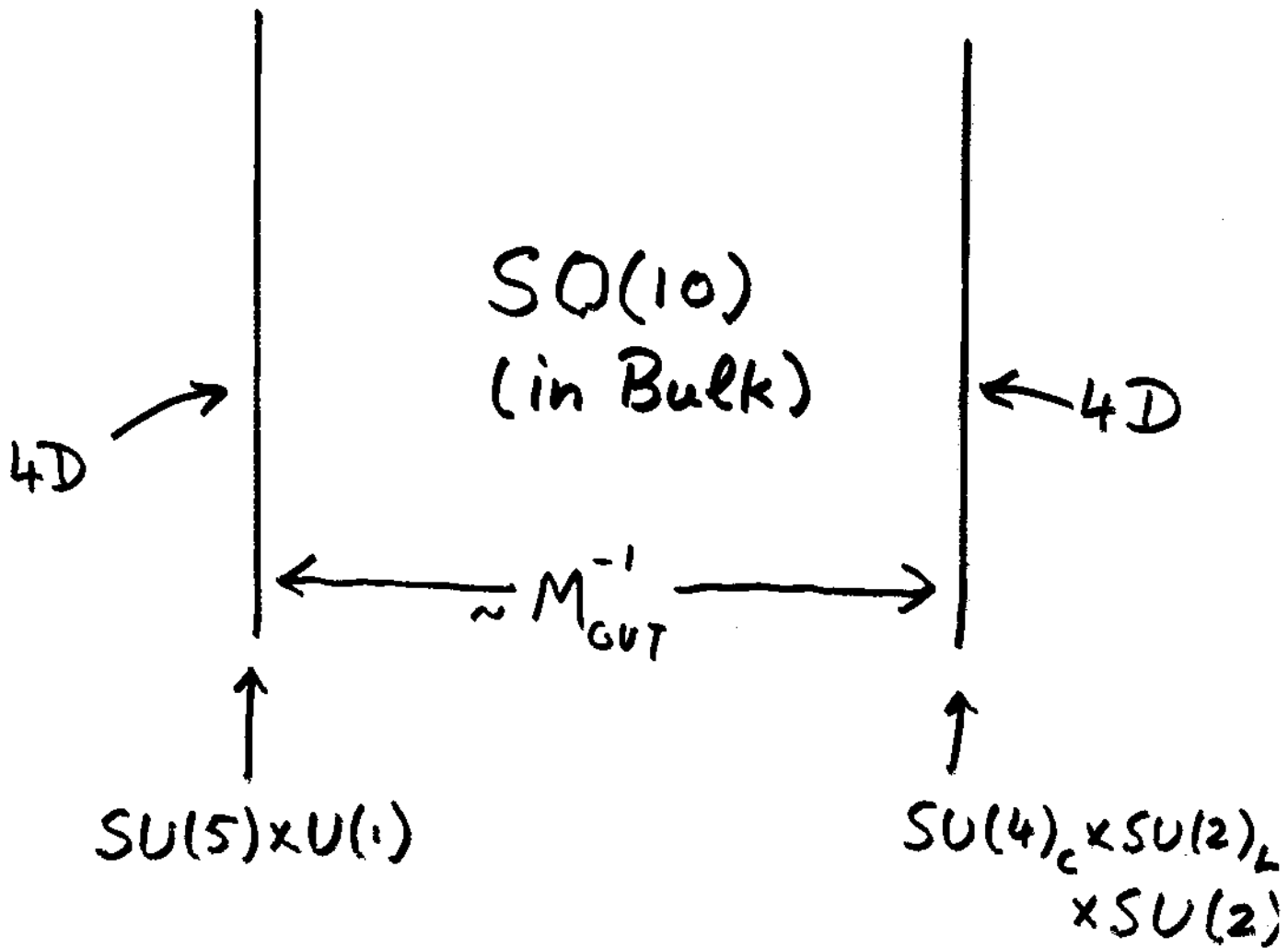
- To suppress it by a significant amount, one can include an additional 45-plet.
- Gauge boson mediated decay  $p \rightarrow e \pi^0$  then becomes dominant ( $\tau_p \sim 10^{35} - 10^{36}$  yrs).

dim 5 p decay





# 5-D $SO(10)$



- 5D  $SO(10)$  compactified on  $S^1/\mathbb{Z}_2 \times \tilde{z}$
- 4D model is  $MSSM \times U(1)$ , with  $N=1$  supersymmetry;  $\hookrightarrow$  break before
- Dim 5  $p$  decay heavily suppressed
- But  $p \rightarrow e^+ \pi^0$  enhanced relative to susy  $SU(5)$  by a factor  $\sim 10-10^2$

# SUMMARY

- MSSM  $\times$   $U(1)_{B-L}$  is a compelling extension for implementing:  
inflation,  $\delta T/T$ , leptogenesis;
  - resolving MSSM  $\mu$  problem;  
stabilizing proton
  - Dim 5  $p$  decay is model dependent; heavily suppressed in some models (including  $SO(10)$ ).
  - $\tau_{p \rightarrow e^+ \pi^0} \sim 10^{34-36}$  yrs in some schemes.
- FIND IT!