XI International Workshop on "Neutrino Telescopes"

Lepton and Quark Masses in SO(10)

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Introduction

We formulate the problem of lepton and quark masses in general terms. We recall an useful tool to discuss the fermion masses in $SO(10)$*, namely, the language of Pati-Salam subgroup.*

The main question

Each family of quarks and leptons fits into the 16_F of $SO(10)$. It is very appealing that quarks cannot exist without leptons. However, this raises an important question:

> **How can we have such quark and lepton unification, when their masses and mixings are known to be so different?**

The questions arises for other unified groups as PS=$SU(4) \times SU(2)_L \times SU(2)_R$ or $SU(5)$, but not for SM=$SU(3) \times SU(2)_L \times U(1)$.

To address this question, we need to define the SO(10) model fully. In particular, we need to define the higgs sector of the theory.

We discuss the features of

1. **Neutrino masses**
2. **Charged fermion masses**
3. **Spontaneous symmetry breaking**

in supersymmetric and non-supersymmetric SO(10) models. We adopt simplicity as a guide, e.g., as few higgs multiplets as possible, and restrict our attention to renormalizable lagrangians only.

Number of non-ren. interactions is rather large, so last assumption is motivated by simplicity, but also by other considerations, e.g., fast proton decay with $1/M_{pl}$ *interactions in supersymmetric theories*

We will see that points 1., 2., 3. are closely connected among them and with other facts, as the expectations on proton decay

FERMIONS IN PS

In Pati-Salam group $SU(4) \times SU(2)_L \times SU(2)_R$ we have

$$(4,2,1) = \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix}_L , \; (4,1,2) = \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix}_R$$

BILINEARS (=YUKAWA COUPLINGS) IN PS

Dirac type couplings

$$\overline{\psi_R}\psi_L = (\overline{4},1,2)\cdot(4,2,1) \left(\begin{array}{c} 1 \\ 15 \end{array} , 2 , 2 \right)$$

since 15 contains a colorless components, both representations can be used as a higgs field

Majorana type couplings

$$\psi_L\psi_L = (4,2,1)\cdot(4,2,1) = \left(\begin{array}{c} 6 \\ 10 \end{array} , \begin{array}{c} 3 \\ 1 \end{array} , 1 \right)$$

10 contains a colorless components; if it has $T_{3L} = +1$ it is also neutral, since $Q = T_{3L} + T_{3R} + (B-L)/2$

HIGGS COUPLED TO FERMIONS IN PS

$$(1,2,2) \;,\; (15,2,2) \;,\; (\overline{10},3,1) \;,\; (10,1,3)$$

In the following, we show that we have all these higgses at our disposal in $SO(10)$. Even more, we will use all them

Implementation

We discuss the need of at least 2 higgs fields to account for fermion masses. We select the $\mathbf{10_H}$ *and the* $\overline{\mathbf{126_H}}$ *and show how neutrino and charged fermion masses are implemented.*

Neutrino masses 1 (or, the need of $\overline{126}$)

In $SO(10)$, the fermions of each family sit in

$$\mathbf{16}_F = \begin{cases} (4,2,1)+(\overline{4},1,2) \text{ under } PS \\ 10+\bar{5}+1 \text{ under } SU(5) \end{cases}$$

The only renormalizable Yukawa couplings 16 16 H are with higgs in the representations

$$\mathbf{10_H},\ \mathbf{120_H},\ \mathbf{\overline{126}_H}$$

The 10 higgs leads to the predictions

$$y_b = y_\tau \text{ (good)}$$
$$y_t = y_{\nu_3} \text{ (bad)}$$

the 2^{nd} is unacceptable. Similarly with 120; these two higgs produce *only* Dirac type couplings.

> In order to make the right-handed neutrinos superheavy we need to use $\overline{126}_H$, that contains the PS field (10,1,3) and/or a singlet of SU(5)

This permits to implement seesaw of type I

Charged fermions (need of another higgs)

With 1 type of higgs only, we cannot generate quark and lepton mixings. At least 2 different $SO(10)$ *Higgs should contribute to fermion masses*

Thus we add the $\mathbf{10_H}$ to the $\overline{\mathbf{126}}$

To see this, we use PS language

$$\mathbf{10} = (1,2,2) + \ldots$$
$$\overline{\mathbf{126}} = (15,2,2) + (\overline{10},1,3) + \ldots$$

Using both VEVs, mixings can be generated, for leptons and quarks have different Clebsch-Gordan factors (since $\langle 15 \rangle \propto$ diag$(1,1,1,-3)$)

The point that needs discussion is: under which conditions both bi-doublets acquire a VEV?

Neutrino masses 2 (or, $\overline{126}$, again)

There is another interesting fragment of $\overline{\mathbf{126}}$

$$\overline{\mathbf{126}} = (10, 1, 3) + (\overline{10}, 3, 1) + \ldots$$

Let us comment on these 2 pieces.
Considering the component of 10 in the "direction of leptons" diag$(0, 0, 0, 1)$:

1) when $T_{3R} = -1$ we have the SM singlet. Its VEV gives mass to right-handed neutrinos
2) when $T_{3L} = -1$ we have a neutral component of a SM triplet. Its VEV gives mass to left-handed neutrinos

> **We buy the ingredient for seesaw of type I and get for free the one for seesaw of $\boxed{\text{type II}}$**

As for the bi-doublets, we still need to discuss the size of the VEV of $(\overline{10}, 3, 1)$: next slide illustrates the point

We show how VEVs of $(15,2,2)$ & $(\overline{10},3,1) \in \overline{\mathbf{126}}$ are induced by a VEV of $(1,2,2) \in \mathbf{10}$.

- **$\mathbf{10} \cdot \overline{\mathbf{126}}^{\mathbf{2}} \overline{\mathbf{126}}^{*}$ contains** $(1,2,2)(15,2,2)(\overline{10},1,3)(10,1,3)$, **thus**

  $$\langle 15,2,2\rangle \sim \frac{\langle 10,1,3\rangle^2 \ \langle 1,2,2\rangle}{m^2_{15,2,2}}$$

  suppression by $(M_R/M_{GUT})^2$**, for** $m^2_{15,2,2} \sim M^2_{GUT}$

- **$\mathbf{10}^{\mathbf{2}} \cdot \overline{\mathbf{126}}^{\mathbf{2}}$ contains** $(1,2,2)^2(10,3,1)(\overline{10},1,3)$, **thus**

  $$\langle 10,3,1\rangle \sim \frac{\langle \overline{10},1,3\rangle \ \langle 1,2,2\rangle^2}{m^2_{10,3,1}}$$

  suppression is M_W/M_R **or more, for** $m^2_{10,3,1}$ **can be as low as the** M_R **scale** $= \langle 10,1,3\rangle$

In supersymmetry the mechanism is different but the result is the same (see below)

Summary on quark and lepton masses

The mass relations that we obtain are, e.g.,

$$\begin{aligned} M_d &= Y_{10} < 1,2,2 >_d \quad &+& \quad Y_{126} < 15,2,2 >_d \\ M_e &= Y_{10} < 1,2,2 >_d \quad &-3& \quad Y_{126} < 15,2,2 >_d \end{aligned}$$

For right and left neutrinos,

$$\begin{aligned} M_{\nu_R} &= Y_{126} < 10,1,3 > \\ M_{\nu_L} &= Y_{126} < 10,3,1 > \end{aligned}$$

Thus, the mass matrix of light neutrinos is

$$M_\nu = -M_D^t M_{\nu_R}^{-1} M_D + M_{\nu_L}$$

Points in favor of type II ctrb. (the 2^{nd} piece):

1. *Hierarchical charged fermion mass matrices become compatible with a non-hierarchical ν mass matrix*
2. *In 2×2 case, one proves that seesaw of type II must be the dominant part of ν mass matrix*
3. *Compatibility with data proved by numerical analyses of 3×3 case (θ_{13} is predicted to be large)*

More on type II seesaw in $SO(10)$

As it is well known, the GUT relations $m_b = m_\tau$ and $m_\mu = -3 \times m_s$ work well, and they are compatible with $SO(10)$ relations:

$$\begin{aligned} M_d &= M_{10} \quad + \quad M_{126} \\ M_e &= M_{10} \quad -3 \quad M_{126} \end{aligned}$$

Now, pay attention to the sum-rule

$$M_d - M_e = 4M_{126} \propto M_{\nu_L}$$

The 'hierarchical part' due to 10-higgs(es) drops down if $b - \tau$ unification holds; we are left with a type II contribution to ν mass matrix that is 'non-hierarchical'

Detailed calculations confirm this argument and the viability of type II seesaw

Models

Till now we worried of fermion masses in $SO(10)$*, namely, of* $SU(2)_L$ *and* $SU(2)_R$ *breaking.*

But $\langle\overline{\mathbf{126}}\rangle$ *breaks* $SO(10) \rightarrow SU(5)$*, no more: we still lack a higgs to break* $SO(10)$ *symmetry fully. This is what we discuss in the next 6 pages*

A supersymmetric $SO(10)$ model

A viable supersymmetric $SO(10)$ model is obtained adding 1 'big' higgs and its renormalizable interactions:

$$\boxed{H\mathbf{=210}}$$

A quite unique feature of this higgs field, is that "it kills 3 birds with 1 stone":

⋆ ⋆ ⋆ *$\langle\mathbf{126}\rangle$ & $\langle\mathbf{210}\rangle$ can break $SO(10)$, since* ⋆ ⋆ ⋆

$$\mathbf{210} = (1,1,1) + (15,1,1) + (15,1,3) + \ldots$$

⋆ ⋆ *The VEV needed to mix $(1,2,2)$ and $(15,2,2)$ arises* ⋆ ⋆
e.g., through the F-term of $(15,2,2)_{\mathbf{126}}$:

$$M\ (15,2,2)_{\overline{\mathbf{126}}} + \lambda\ (15,1,3)_{\mathbf{210}}\ (1,2,2)_{\mathbf{10}} + \ldots = 0$$

⋆ *The VEV needed for type II seesaw arises e.g., through* ⋆
the F-term of $(10,3,1)_{\mathbf{126}}$:

$$M\ (\overline{10},3,1)_{\overline{\mathbf{126}}} + \lambda\ (\overline{10},2,2)_{\mathbf{210}}\ (1,2,2)_{\mathbf{10}} + \ldots = 0$$

For last 2 points, characteristic of supersymmetry, we used the term $W = \lambda$ 10 126 210 and the condition $F = 0$

The model with 10, 126, $\overline{126}$, 210 has a number of

$$\textit{supersymmetric parameters} = 26$$

same as MSSM with R-parity assumed and dim-5 operator for ν masses. This number in a renormalizable $SU(5)$ model is ≥ 39.

- *It is possible to solve exactly vacuum condition for:*

$$SO(10) \to SM$$

suggested by 1-step gauge coupling SUSY unification

- *Masses of heavy particles known: e.g., $SU(3)$ triplets that rule proton decay, or $SU(2)$ triplet that controls type II seesaw*

- *Light-higgses emerge upon fine-tuning the doublet's mass matrix. As needed, (1,2,2) and (15,2,2) are both present*

- *A systematic study of implications (proton decay, gauge coupling unif., leptogenesis, LFV) is in progress*

- *However, the model is quite restrictive, up to the point that addition of new higgses could be eventually needed*

A non-supersymmetric $SO(10)$ model

- If SUSY is absent or realized at M_{Pl}

$$SO(10) \to SU(5) \to SM \;\; \textit{is not viable}$$

and the breaking chains should be e.g.:

$$SO(10) \to PS \times P \to PS \to \left\{ \begin{array}{c} LR \\ \\ G_{421} \end{array} \right. \to G_{3211} \to SM$$

some intermediate steps could be contracted, and 'left-right parity' P could be broken elsewhere

- For any chain, the 1^{st} step is to select one (or more) $SO(10)$ higgs fields that permits to realize it

- A convenient starting point to explore the mass spectrum is the 'extended survival principle':

> *the only higgses that are massless above a gauge scale are those needed to break that symmetry (or, higgses take the largest possible mass; or, 'anomalously light higgses' are forbidden)*

What we ask from our model:

- *we want gauge coupling unification;*
- *we explain 2nd family masses by arranging $\langle 15, 2, 2 \rangle$ large enough;*
- *we should not overdo with ν masses (=lower bound on the scale of $SU(2)_R$ breaking);*
- *the proton should live long enough.*

At this point, we study all the possible cases in order to answer the questions:

Which are the breaking chains for which these conditions are possible?

Which other higgses we still need/want for a complete $SO(10)$ theory?

As with SUSY, one begins with $\overline{\mathbf{126}}$ *and* $\mathbf{10}$. *In SUSY, the* $\mathbf{10}$ *is complex, but now we could have a real one. However, this would be insufficient to account for lepton and quark masses! We need at least a pair of real* $\mathbf{10}$*s or better a complex one.*

Now, a surprise from $SO(10)$ symmetry breaking:

The only interesting cases are the variants of a model with a single intermediate scale:

$$SO(10) \to PS \times P \to SM$$

This needs the addition of only 1 new higgs, the

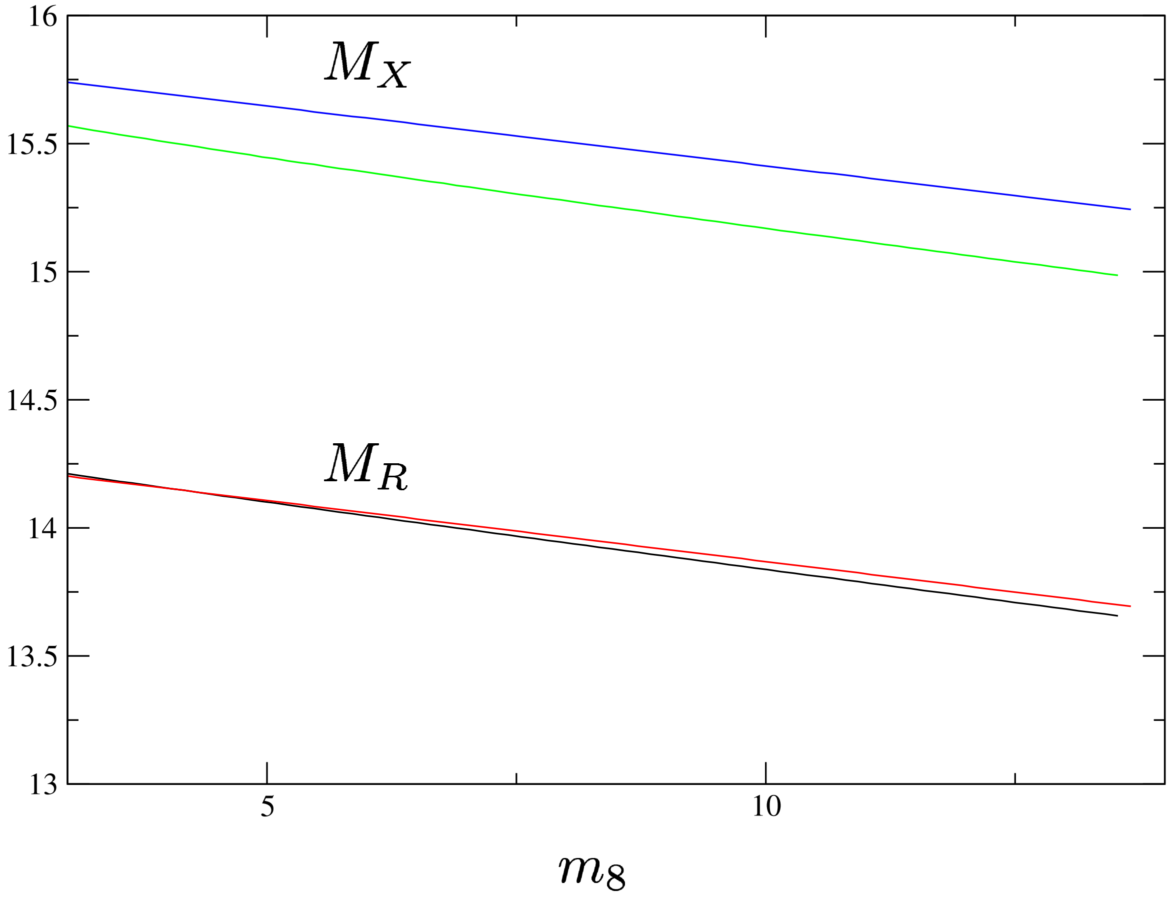
$$\boxed{H = \mathbf{54}}$$

With extended survival principle, the scale of proton decay is a factor of ~ 3 *too small. This is avoided with certain violations of this principle ('anomalously light higgses') or with more intermediate gauge scales near to* $PS \times P$*. In view of minimality,* 1^{st} *option is most attractive*

We emphasize:

the indication for 'rapid' proton decay rate;

that type II seesaw can be the dominant mechanism for neutrino masses.



Plot of the grand unification mass M_X *and of the scale of* $SU(2)_R$ *breaking, as a function of the mass of the 'anomalously light higgs'.*

Masses are expressed in log-form: for instance, 16 means $M = 10^{16}$ GeV

Effect of 2 loop is shown to lower M_X

Summary and discussion

- *Lepton and quark masses give precious indications to formulate a complete $SO(10)$ theory.*

- *We decided to emphasize the role of minimality and of the gauge principle (thus we avoided recourse to non-$SO(10)$ concepts, such as flavor symmetries, $1/M_{Pl}$ contributions, ad hoc discrete symmetries such as R-parity in SUSY, etc.).*

- *Some minimal cases seem to emerge. However, minimality and gauge invariance alone are not enough to select a single option. In particular, they are unable to decide between SUSY and non-SUSY.*[†]

- *Some tests of these ideas are already known (say, with leptonic mixing angles), other ones are quite natural (e.g., those related to proton decay), and we would be not much surprised if other important tests just wait for a discovery.*

- *Perhaps, we do not really need new views, but just new efforts on old good $SO(10)$.*

Thank you for the attention!

[†] Thence, the name we have chosen for our collaboration: A.Mi.G.O.= Another Minimal GUT, Oh God!

Some references

- *Pati-Salam group (leptons as the 4^{th} color) has been introduced in 1974.*
- *SO(10) has been first proposed one year later by Fritzsch & Minkowski and independently by Georgi.*
- *The importance of the $\mathbf{126}$ was realized in 1979 by Glashow and Gell-Mann, Ramond, Slansky*
- *(A very useful work for any model building is the Ph.Rep. of Slansky, 1981)*
- *The contribution of $\overline{126}$ to charged fermion masses was first noted in Lazarides, Shafi, Wetterich (NPB 1981). It was adopted and emphasized in a PRL of Babu & Mohapatra of 1993. (Note the similarities of this $SO(10)$ mechanism with the one suggested for $SU(5)$ by Georgi & Jarlskog, 1979).*
- *The possibility of type II seesaw in our version of $SO(10)$ was noted in 1980 by Magg & Wetterich (PLB) and by Mohapatra & Senjanović (PRD). The usefulness of this seesaw in connection with non-hierarchical matrices (with large mixing angles and weak mass hierarchies) was remarked and discussed by Bajc, Senjanović, FV in hep-ph/0110310, in PRL (2002), and in PRD (2004). Existing 3 flavor analyses are due to Goh, Mohapatra, Ng (2003). An analysis in progress by Bertolini & Malinsky is confirming the viability of the model, however with wider parameter space*

- The supersymmetric model was introduced independently by Clark, Kuo & Nakagawa (PLB, 1982) and Aulakh & Mohapatra (PRD, 1982). The model has been further discussed by DG Lee (PRD 1993) and recently by Aulakh et al (PLB 2004). In the last paper, emphasis was put on fermion masses
- After Aulakh et al (PLB 2004) the need of a detailed calculation of the mass spectrum was realized. This was performed by Aulakh et al (hep-ph/0204097 V4), Bajc et al (PRD 2004), Fukuyama et al (J.Math.Phys. 2004).
- A claim of the last group, that the other calculations were inconsistent with their one, is shown to be immaterial in Aulakh hep-ph/0501025; compare also V1 and V2 of hep-ph/0412348.
- Some of these groups and other ones presented the explorations of the predictions of the model. Claims have been done that addition of new particles is necessary, see J Sato (PRD 1995); Goh, Mohapatra, Nasri (PRD 2004). See also Mohapatra, this workshop
- Finally, about non-supersymmetric model: we are not sure which is the first paper where the model with 54 is proposed (we found discussions as early as in Li-fong Li, 1974) However, relevant recent papers should include the series of works of Buccella and collaborators (e.g., Abud et al 1986) and 2 PRD's by Deshpande, Pal, Keith (1992). Most results presented here, however, are original. They are based on a work our collaboration will publish soon (2005)